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THE EFFECT OF DYNAMIC MATHEMATICS LEARNING ENVIRONMENT ON THE SOLO UNDERSTANDING LEVELS FOR EQUATIONS AND INEQUALITIES OF 8TH GRADERS

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Abstract

Dynamic mathematics software allows students making experiments to discover mathematical concepts on their own. In this way, the mathematical understanding can be supported more lasting and effective way. In this study, it is aimed to determine the mathematical understanding of 8 graders on equations and inequalities in the dynamic mathematics software supported learning environment in terms of the Structures of the Observed Learning Outcomes (SOLO). A quasi-experimental design was implemented. The experiment was designed as pretest-posttest without control group and supported by qualitative data. A group of 18 8th grade students studied equations and inequalities during 4 weeks. In the learning environment, the dynamic mathematics software GeoGebra was used in the view of constructivist philosophy, especially students were guided to make trial and error by taking advantage of different representations. Student's mathematical understanding level was measured by the conceptual understanding level pre and posttests (pre-test and post-test) which include open ended questions. The pre-test and post-test were assessed and scored in terms of the SOLO taxonomy. Besides, students' pre-test and post-test answers were analyzed qualitatively by content analysis method. At the end of the study, although the scores of students' pre-test and post-test were not significantly different, they showed better performance in post-test. Furthermore, it is observed that students were tending to use multiple representations in post-test in the qualitative analysis.

Key Words: *Multible Representation, Dynamic Mathematics / Geometry Softwares, Equations and Inequalities, GeoGebra, SOLO Taxonomy.*

DİNAMİK MATEMATİK YAZILIMI DESTEKLİ ÖĞRENME ORTAMININ 8. SINIF ÖĞRENCİLERİNİN DENKLEM VE EŞİTSİZLİKLER ÜZERİNDEKİ SOLO ANLAMA SEVİYELERİNE ETKİSİNİN İNCELENMESİ

Özet

Dinamik matematik yazılımları öğrencilerin deneme yanılmalar yapabileceği ortamlar sunarak matematiksel kavramları kendi kendilerine keşfetmelerine imkân tanımaktadır. Bu sayede matematiksel anlamanın daha kalıcı ve etkili şekilde desteklenebileceği söylenebilir. Bu çalışmada, matematiksel anlamayı ölçme yöntemlerinden biri olan Gözlenen Öğrenme Çıktısı Yapıları (Structures of Observed Learning Outcomes-SOLO) sınıflandırmasına göre dinamik matematik yazılımı destekli ortamda matematik öğrenen öğrencilerin matematiksel anlamalarının nasıl etkilendiğini belirlemek amaçlanmıştır. Yarı deneysel bir desen olan tek grup öntest-sontest kontrol grupsuz desene göre tasarlanan ve nitel veriler ile de desteklenen bu çalışmada 18 adet 8. sınıf öğrencisinden oluşan bir grup 4 hafta boyunca denklem ve eşitsizlikler konusunu çalışmışlardır. Öğrenme ortamında, dinamik matematik yazılımlarından GeoGebra yapılandırmacı bir felsefe ışığından kullanılmış, özellikle öğrencilerin farklı temsillerden yararlanarak deneme yanılmalar yapmalarına rehberlik edilmiştir. Açık uçlu sorulardan oluşan ön Kavramsal Anlama Tespit Sınavı (ön-test) ve son Kavramsal Anlama Tespit Sınavı (son-test) yardımı ile öğrencilerin matematiksel anlama seviyeleri SOLO sınıflandırmasına göre belirlenmiştir. Ayrıca, öğrencilerin ön-test ve son-test cevapları içerik analizi yöntemi ile analiz edilerek cevaplardaki öğrencilerin üzerinde durdukları temel noktalar belirlenmeye çalışılmıştır. Çalışmanın sonunda, öğrencilerin ön-test ve son-test solo anlama seviyeleri arasındaki farkın parametrik olmayan testlerden wilcoxon işaretli sıralar testi ile incelenmesi neticesinde anlamlı bir fark bulunmamasına rağmen *öğrencilerin SOLO anlama seviyelerinin bir parça artış gösterdiği ve son-test cevaplarında farklı temsilleri bir arada kullanma eğilimi sergiledikleri gözlemlenmiştir. Bu sonuç, farklı temsiller ışığında öğrenme tecrübesini hiç yaşamamış öğrencilerin matematiksel anlama seviyelerinin doğrudan etkilenmeyebileceği, ancak farklı temsilleri kullanabilme alışkanlığı kazanmaya başladıkları şeklinde yorumlanmıştır.*

Anahtar Kelimeler: *Dinamik Öğrenme, Çoklu Temsil, Dinamik Matematik/Geometri Yazılımları, Denklem ve Eşitsizlikler, GeoGebra, SOLO Taksonomisi*.

Introduction

Technology has streamlined the transition from traditional education to student-centered education by adding different dimensions to the education process. Information technologies should be used as a bridge that features students' knowledge and skills, and offers interactive learning opportunities rather than merely as a tool that enlivens the traditional education processes as a calculation, graph plotting or presentation tool (Baki, 2002). Multiple representations represent a mathematical concept or relation with an equation, formula, graph, table, figure or symbol. Using these representations in mathematics education facilitates seeing mathematical concepts in different ways, and establishing connections among them (Even, 1998; Hiebert & Carpenter, 1992). The use of technology aids in developing the students' mathematical concepts and skills, dealing with problems, developing visualization skills, and extending mathematical knowledge via multiple representations. Thanks to one of the technological tools, dynamic geometry software, the students structure the acquired knowledge and the theorems they were taught rather than memorizing them (Karatas & Guven, 2003).

In the research literature, it was shown that dynamic software-aided multiple representations have a positive effect on students' learning and their success in learning mathematics (Abdusselam, 2006; Akkus & Cakiroglu, 2006; Filiz, 2009; Ozgun Koca, 2004; Sireci, 2004; Tiryaki, 2005). It was also observed that computer technology affected students' attitudes towards mathematics positively via the learning environment it offered (Ozdemir & Tabuk, 2004; Sulak, 2002). Moreover, it was observed that dynamic mathematics software directs students towards mathematical concepts (Ozgun Koca, 2004), in that it allows students to make deductions, and directs them to conceptual understanding by avoiding memorization (Filiz, 2009; Sireci, 2004). Furthermore, it provides a constructivist learning environment by enriching the mathematics learning environments (Chrysanthour, 2008).

Sometimes, the effect of using technology on mathematics success was measured by examinations involving multiple choice questions only (Kilic, 2007; Sulak, 2002; Unlu, 2007). Although these and similar studies showed the positive effects of technology, it can be inferred that they provide clues about better memorization of mathematical concepts, rather than understanding the concepts. Open-ended questions, which enable students to express themselves, better, may be hard to score. On the other hand, open-ended questions are incomparably more advantageous than multiple choice questions, by measuring the conceptual knowledge in more detail and rendering the methodology of solving questions (Henningsen & Stein, 1997). In this context, investigating the effect of using dynamic mathematics software needs to be held by open-ended measurement tools more. The SOLO (Structures of the Observed Learning Outcome) taxonomy is the model that is employed particularly to assess the students' knowledge and skills in learning environments (Biggs & Collis, 1991). The SOLO model enables classifying the students' answers to certain stimuli in terms of their quality and structure. The understanding levels of the students are explained in 5 stages (Biggs & Collis; Pegg & Tall, 2005). These stages are summarized in table 1. Mooney (2002) and Rider (2004) also used the classification above to determine a quantitative scoring of one's SOLO level. They give the scores from 1 to 5 as from pre-structural to extended abstract levels respectively.

Table 1: *SOLO Understanding Levels*

SOLO Understanding Levels	
Extended abstract (The highest level)	Student can reason by taking into account the abstract features and can generalize structures.
Relational	Student understands all aspects of the answer, their place within the whole, and the relations among each other.
Multi-structural	Student can utilize more than one aspect, situation, concept or structure regarding the answer. However, s/he cannot establish relations among them.
Uni-structural	Student focuses on one single aspect of the problem. S/he cannot relate the part to the whole. His/her answers are not consistent.
Pre-structural (The lowest level)	Student is not interested in the task given or has very little understanding about the task.

Explicit evidence also has been seen in the literature saying that the SOLO taxonomy is a strong and efficient tool to measure the student's understanding levels for various subjects qualitatively (Akkaş, 2009; Çelik, 2007; Groth & Bergner, 2006; Lian & Idris, 2006).

The subject of equations and inequalities was deemed as a subject fit for this study since it involves various mathematical representations. The studies about teaching the concepts of equations and inequalities showed that students have inadequacies in understanding the relations among different representations of these concepts (Akkan, Cakiroglu, & Guven, 2009; Pomerantsev & Korosteleva, 2003).

In conclusion, the purpose of this study to analyze the effects of dynamic mathematics software-aided mathematics education on the conceptual understanding levels of the 8th grade students on the subject of *equations and inequalities* in terms of the SOLO taxonomy. The research will try to answer the question "Does the dynamic mathematics software positively affect the 8th grader's SOLO understanding levels?" and it is important both investigating the effect of technology qualitatively and increasing the number of studies on using SOLO taxonomy in the literature.

Method

Since the objective of this study is to analyze the effect of dynamic mathematics software-aided mathematics education on the conceptual understanding levels of the 8th grade students on the subject of equations and inequalities in terms of SOLO taxonomy, a research pattern that involves both qualitative and quantitative data analysis was designed. This study was formally conducted as quasi-experimental design that is called *pretest-posttest without control group model*. Initial and final measurements were employed to compare the students' initial conceptual understanding levels and their final conceptual understanding levels. The difference among the scores that are designated according to SOLO taxonomy and that indicate the conceptual understanding levels were quantitatively analyzed using a nonparametric statistics test called the "Wilcoxon signed-rank test". The students' initial measurement and final measurement examination papers were assessed qualitatively using the content analysis method in order to determine the frame in which the change in conceptual understanding levels is realized.

Research Group

This study was implemented in a primary school located in a town around the Denizli city center during the 2011 fall semester. The unique 8th grade class of the school was determined as experimental group. The socioeconomic level of this primary school, which generally consists of students that have average success in mathematics, is generally around low to medium. There are an equal number of male and female students in the class determined as the experimental group.

Treatment Process

In this study, the education process was implemented in a manner progressing from graphical representation to algebraic representation, provided by the dynamic mathematics software use. The class environment was arranged so that every two students had access to one computer. The topic of equations and inequalities was conducted using GeoGebra for 4 weeks, 6 hours per week under the guidance of pre-prepared worksheets. These activities were designed to enable the students to discover the discussed concept using the GeoGebra software, and to establish their own learning paths. In order to better explain the basic philosophy underlying the implementation process, the implementation of the activity regarding inequalities with one unknown is described below.

Contrary to the prevalent traditional approach, the students were not asked to consider inequalities first as equalities/equations to find a solution and to state the solution sets accordingly using the knowledge provided by the teacher. The students were made to consider the right and left sides of the inequality $2x + 1 < 1 – 3x$ and similar inequalities as a separate line equation and obtain their graphs using the software (they were asked to utilize their previous knowledge since they studied the solution of the equations using the graphical approach before). Naturally, since a different notation was used, they initially had difficulty in understanding. At this point, the teacher provoked their reasoning by asking the question “Could you find the x values for which the line $2x + 1$ (left hand side of the inequality) had smaller values than the line $1 – 3x$ (right hand side of the inequality)?” By this way, the students were made to realize that they should find the intersecting point first, in order to find the x domain where one of the intersecting lines is under the other. The students established the necessary solution procedure for different inequality cases themselves sometimes by using pen and paper first, then the graphical approach and sometimes following the reverse order.

Data Collection

The conceptual understanding level pre-test and conceptual understanding level post-test were given in order to determine the change in the students' conceptual understanding levels before and after the implementation.

Since the goal was to determine the students' conceptual understanding levels of a specific mathematical topic rather than their success in that topic, the pre-test, which measures their conceptual understanding levels, involves an equations topic that the students learned in the 7th grade as a part of the algebra curriculum. On the other hand, the post-test questions involve the equations and inequalities topic of the 8^{th} grade that was taught during the implementation process. The pre-test and post-test questions were prepared so that the students' conceptual understanding in the mentioned topic could be determined by taking advantage of the field experts' opinions and the literature. The opinions of the experts were recorded by using a form that contains the possible answers that could be given for each question and the predictions about how they reflect the conceptual understanding. The examinations, whose scope validity was ensured this way, were given in their final form as 3 questions for the pre-test and 4 questions for the post-test (see appendix).

Implementing the Tests: The pre-test, which determines the students' initial conceptual understanding levels, was given during one class hour after their 7th grade knowledge was revised. The decision was made to allow the students to take advantage of the dynamic software during the post-test that was given after the dynamic software-aided education provided during the implementation process, in which two students had access to one computer. The class was divided into two 9-student groups in order to prevent students from assisting each other, and they were given the examination in two sessions. The group that took the examination first was prevented from communicating with the other group.

Data Analysis

The pre-test and post-test answers were scored by enumerating five SOLO levels, which were previously explained in Table 1, from 1 to 5. The blank answers or the answers where random operations were performed with the numbers in the question were assigned to the *prestructural level* (1 point). The answers where a single aspect of the problem was dealt with and where operations were performed in parts but could not be combined were assigned to the *unistructural level* (2 points). The answers where every step towards the solution of the question was taken but could not be finalized were assigned to the *multistructural level* (3 points). The answers where the questions were understood in all aspects, solved, and explained in any way were assigned to the *relational level* (4 points). The answers where different representations were used in combination for the solutions, and the results were verified by crosschecking were assigned to the *extended abstract level* (5 points) (Mooney, 2002; Rider, 2004).

The significance of the difference among the pre-test and post-test scores was calculated in accordance with the scoring system explained above and analyzed by the Wilcoxon signed-rank test. In order to deeply analyze the students' answers and to determine the focal elements of the answers, the pre-test and post-test answers were put through a content analysis by two different coders that were graduate students in mathematics education and one was the second author. The difference between the pre-test and post-test was also attempted to be presented qualitatively by determining the common codes and themes.

Results

The findings of this study were analyzed in two (quantitative and qualitative) stages. The analysis of the quantitative data is given first.

Quantitative Analysis

Since the research group lower than 30, the scores were analyzed by a nonparametric test called the Wilcoxon signed-rank test. Nonparametric statistical significance tests make inferences using quartile values of the data group instead of values such as the average and standard deviation; therefore, the following table, where the group's descriptive statistics are given, contains the quarter values in addition to the average and standard deviation.

Table 2: *The Descriptive Comparison of the Pre-test and Post-test*

	N	Average	Standard Deviation	Lower quartile	median	Upper quartile
Pre-test	18	2.12	1.18	1.00	1.70	2.78
Post-test	18	2.46	1.15	1.50	1.90	3.58

When the data in table 2 is examined, the students' post-test scores averages (2.46) are found to be higher than their pre-test scores averages (2.11) and the standard deviation values did not change much. Also a quick look at the quartile values shows that the post-test values are higher and the slight increase in the lower quartile and median values are higher for the upper quartile value. This situation is shown visually in figure.

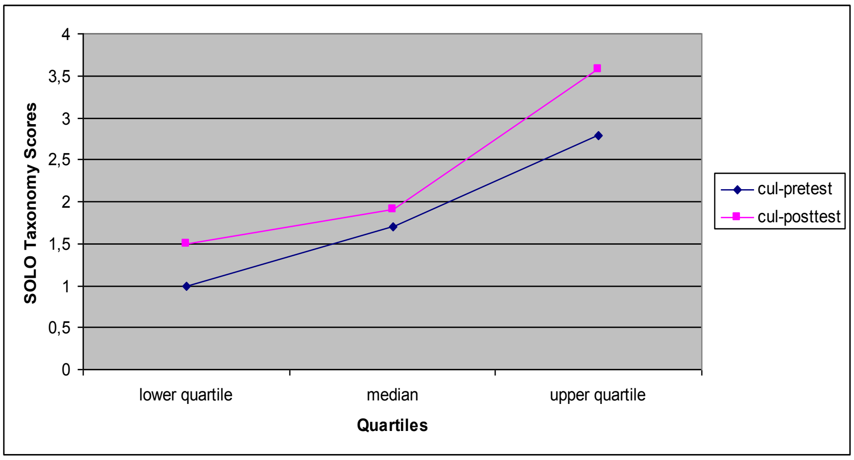


Figure 1: *The Comparison of Lower, Median and Upper Quarter Values of the Pre-test and Post-test Scores*

Especially the increase of the upper-quartile closing the 4th Solo level is remarkable here. In other words, an increase is observed in the students' conceptual understanding levels in the upper quartile. Therefore, it can be said that the students' understanding level tends to rise to the relational level, which is the 4th level in the SOLO taxonomy, after the course given in a learning environment enriched with multiple representations, supported by the dynamic mathematics software.

The analysis results given above show that the dynamic mathematics software-aided mathematics education changes the students' conceptual understanding levels positively. In order to determine if this change is significant, the difference between the students' pre-test scores averages and post-test scores averages were analyzed. The findings obtained as a result of the analysis are shown in table 3.

Table 3: *Wilcoxon Signed-rank Test Results for Pre-test and Post-test Scores*

Cul-posttest – Cul-pretest	N	Rank Average	Rank Total	Z
Negative Rank	5	7.30	36.50	
Positive Rank	11	9.05	99.50	-1.63*
Equal	2			

p> .05

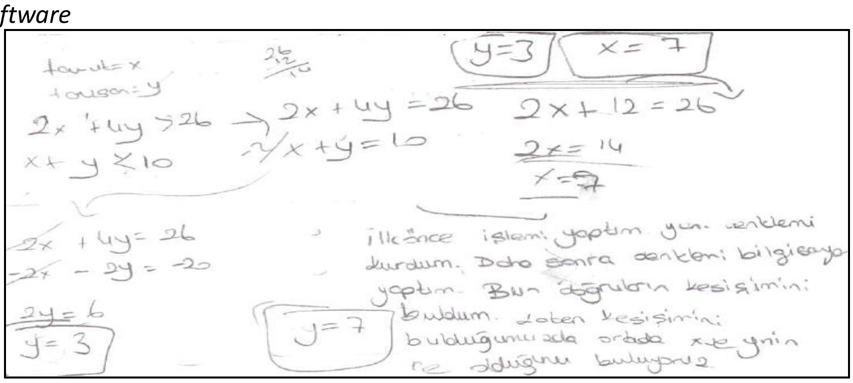
As seen in table 3, although there are positive ranks in the students' pre-test and post-test scores (the results are in favor of the post-test) this difference is found to be an insignificant change (z = -1.63, *p* > .05). Based on the analysis of the data presented above, it can be said that the dynamic mathematics software-aided mathematics education creates a positive difference in the students' conceptual understanding levels; however, this difference is statistically insignificant. This result may seem surprising due to the expectation of significant changes in the students' conceptual understanding levels as a result of a more innovative education. However, the lack of significant change in the mathematical understanding levels of the students, who never experienced learning in light of different representations, after a one-month course can be considered normal. At this point, the qualitative analysis of the students' answers may facilitate the acquisition of a more explanatory result.

Qualitative Analysis

The students' answers to the pre-test questions were completely focused on an algebraic solution. It was observed that the basic factor determining the students' levels was whether they were able to finalize the algebraic solution attempt or not.

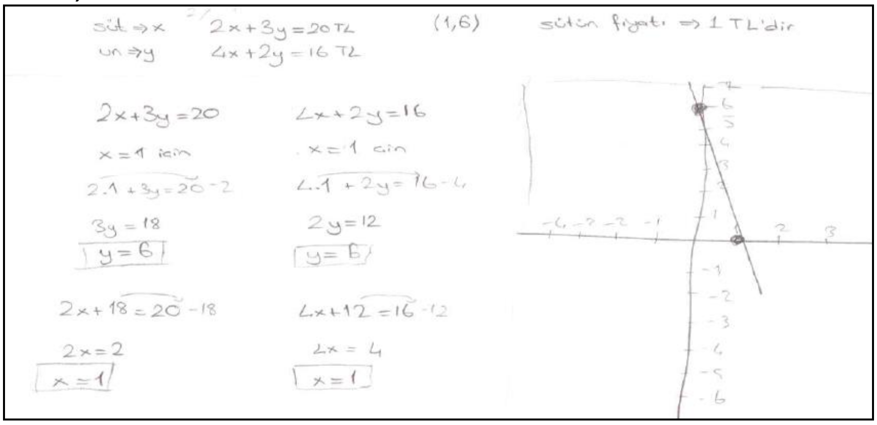
On the other hand, the answers to the post-test questions involved more varied answer types. The post-test answer illustrated in figure 2.1 shows that the student satisfied his/her expectation to observe the algebraically found answer on the graph acquired with the software, as well. Students wrote a note as an explanation "*I made the operation first, then I checked it on graphic and saw that the intersection point is exactly same with the point that I calculated previously.*"

Figure 2.1. *The Answer of a Student that Achieved the Graphical Representation via Software*



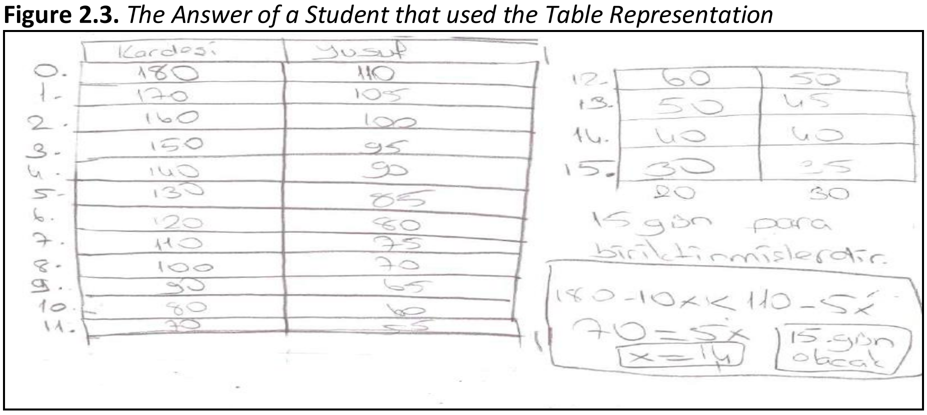
Another post-test answer given as an example in figure 2.2 shows that the student attempted to use a representation other than the algebraic representation by plotting the graph on the answer paper, albeit incorrectly. Although the incorrect graph plotted by the student negatively affected the scores s/he received for this question, in fact it reflects an attempt to study the solution of the question from a different perspective.

Figure 2.2. *The Answer of a Student that Plotted the Graphical Representation Manually*



Moreover, it was observed that some students used the table representation while solving the post-test questions (figure 2.3). Unlike previous examples, this student searched for a solution to the question by using a different representation, namely the table representation, instead of crosschecking an algebraic solution using a different representation.

Figure 2.3. *The Answer of a Student that used the Table Representation*



The result summarized in table 4 was achieved by analyzing all students' answers in terms of using multiple representations and by taking into account the case of using more than one representation at least in one answer to a question.

Table 4: *The Qualitative Analysis of the Students' Answers*

	Pre-test	Post-test
Using single representation	18	12
Using multiple representation	0	6

In summary, it was observed that in the answers to the pre-test questions, that all students attempted to solve the questions by using algebraic representation only, and in the answers to the post-test questions, 6 students approached at least one question by also using a different representation. Moreover, it also was observed that although the students were free to use computers during the post-test, the students did not abuse this opportunity; they did not try to produce meaningful/meaningless results via the computers, and only used the computers to show their attempts to produce different representations. This situation can be assessed as proof of students' tendency to use the software as a conceptual aid rather than as a calculator.

Discussion

As a result of the implementation of teaching equations and inequalities in a dynamic mathematics software-aided learning environment where multiple representations are highlighted, the test group's pre-test and post-test averages and their answers in these examinations were compared.

As a result of the analyses performed, the students' test averages increased after the implementation although a statistically significant difference was not observed between the students' post-test averages and pre-test averages. The actual important result of this study was achieved by analyzing the students' answers qualitatively in a more detailed manner. It was found that the students began to reflect a tendency to use the table and graph representations as they were finding their answers. This finding is parallel to the studies that highlight the positive effect of the multiple representations on learning (Abdusselam, 2006; Akkus & Cakiroglu, 2006; Filiz, 2009; Ozgun-Koca, 2004; Sireci, 2004; Tiryaki, 2005). In particular, Ozgun Koca (2004) stated that the dynamic mathematics software directed students to conceptual understanding, and this study also showed that the students' answers after the implementation were directed towards studying the conceptual structure. Furthermore, it was observed that some students used the dynamic mathematics software to produce a graphical representation during the post-test. Their use of the software to satisfy their need to observe their algebraically obtained results via a different representation rather than using it as a calculator to streamline the calculations, reminds us that the software reinforces conceptual learning (Filiz, 2009; Sireci, 2004), and that the relations among different representations reinforces learning (Even, 1998; Hiebert & Carpenter, 1992).

Despite the strong opinion in the literature regarding the dynamic software's role in highlighting conceptual learning, particularly regarding multiple representations being one of the important components of meaningful learning, in this study the change in the students' SOLO levels were not significant, which may have many explanations. Among these reasons, the most important is that there was a short period of time (4 weeks) between the initial measurement and the final measurement. Taking into account the fact that the students' learning culture, particularly in Turkey where it has been established that information is relayed in a classic manner, the students may have various difficulties in adapting to this new learning culture in a short period of time. In the studies where the effect of the dynamic mathematics software-aided learning environment was surveyed, the comparisons are mostly based on multiple choice achievement tests and the results were generally significantly in favor of the groups that studied using the dynamic mathematics software (Kilic, 2007; Sulak, 2002; Unlu, 2007). When choosing the correct answer instead of a conceptual solution effort is designated as the criterion for success, the achieved results can in fact be considered as a natural consequence of the use of technology causing positive affective development (Ozdemir & Tabuk, 2004; Sulak, 2002). On the other hand, this study attempted to determine the effect of the dynamic mathematics software on success in accordance with the SOLO taxonomy. The effect of the dynamic mathematics software on conceptual learning and success, which was partly apparent in this study, would be unveiled more clearly by studies that involve more detailed surveys.

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Appendix

Conceptual Understanding Level Tests (Cul-Tests)

Conceptual Understanding Level Pre-test

1. A group of tourists drink at cup of tea in a café where the price of a cup of tea is 2,5 TL. Since 4 of them do not have enough coins, all others pay 0,5 TL more. How many tourists are included in the group?
2. Ali visited 20 of his relatives in the Ramadan holiday. Some of his relatives gave 20 TL to him and remaining ones gave 10 TL. If Ali has 280 TL at the end of the all visits, how many relatives gave 10 TL to Ali?
3. Find the solution set of the equation $5x+24=89$.

Conceptual Understanding Level Post-test

1. Ahmet and his brother aimed to save money to buy a toy in the summer holiday. They decided to put a coin into moneybox from their wallet. At the beginning, 18 coins of 10 Kurus existed in the Ahmet's wallet while, 22 coins of 5 Kurus existed in his brother's wallet. After some days, they see that Ali's money is less than his brothers' money. So, can you find the number of days that two brothers save money?
2. Solve the inequality $1-2x<5$.
3. The total number of legs of chickens and rabbits is greater than 26. If the total number of chickens and rabbits is lower than 10, how many chickens and rabbits may exist?
4. 2 liter of milk and 3 kg of flour costs 20 TL while 4 liter of milk and 2 kg of flour costs 16 TL. Find the price of 1 liter of milk?