LEARNING ABOUT THE BULLWHIP EFFECT USING COLORED PETRI NET SIMULATOR

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ABSTRACT: Colored Petri Net (CPN) simulator for study and analysis of supply chain processes is described in this paper and some experimental results obtained by its use are presented. This CPN simulator is one version of the well-known beer game and overcomes some shortcomings we have noticed playing other beer games. The beer game is a role-play simulation game that lets students (or managers as well) experience typical coordination problems of supply chains. Supply chain consists of four stages (or co-makers): retailer, wholesaler, distributor and manufacturer. The simulator allows calculation of different supply chain performances. It is developed using a timed, hierarchical colored Petri Net and CPN Tools software package. The CPN structure developed and presented in this paper consists of one top page and 17 sub-pages in four hierarchical levels, which models decomposition of observed supply chain in sub-processes. For each stage, demand forecasting methods, replenishment policies, production and delivery times, inventory costs, and customer’s demand may be defined and given as input data for simulation run. In CPN Simulator we have used three strategies for demand forecasting. First strategy is based just on the past demand and second and third strategies are based on adaptive time series method: Moving Average and Exponential Smoothing Methods. We conducted three groups of experiments, every group for a different strategy of demand forecasting and for every group we experimented with several different parameters. The simulation results are exported to Excel and its visual presentation and expressive reporting capabilities are used. This package is aimed for evaluation of different management strategies in a supply chain, as well as for educational purposes. This developed model can efficiently be used for teaching the bullwhip effect not only to undergraduate and graduate students, but managers as well.

Key words: Supply chain, bullwhip effect, petri net

INTRODUCTION

Very important supply chain (SC) processes are order and delivery of purchased amounts. These are multiple entangled and their disorder can lead to various unwanted effects. One of them is the bullwhip effect. Different chain phases have different calculations on demand quantity, thus the longer the chain between the retailer and wholesaler the bigger the demand variation. A retailer can realize a small variation in customers’ demands as a growing trend and purchase from wholesaler more products than he needs. An increased order at wholesalers is larger than at retailers as the wholesaler cannot regularly comprehend the increased order. As the chain grows longer the order is larger. If a retailer plans the product promotion he can increase the order. If a manufacturer comprehends the increased demand as constant growth and in the same manner makes purchases, he will face the problem of inventory surplus at the end of promoting period, (Chopra S. and P. Meindl, 2001). Variation of demands increases production expenses and expenses of the whole supply chain in an effort to deliver the
ordered quantity in time. A manufacturer accomplishes demanded capacity and production but when the orders come to a former level, he remains with the surplus of capacity and inventory.

Originally, beer game was created as a board game that demonstrates beer production and distribution as shown in Figure 1, but now there are several electronic games for simulating Beer Game (Li M. and D. Simchi-Levi, 2002; http://beergame.masystem.se:8000/; http://www.forio.com/nearbeer.htm). However, all electronic games for simulating Beer Game we have found allow experiments with the strategies of one SC participant per game. Because of that fact our motivation was to design simulator that allows one participant to simultaneously experiment with the parameters of all SC stages. That type of simulator we have developed using hierarchical Coloured Petri Nets (CPN).

Coloured Petri Nets are a modelling language developed for systems in which communication, synchronization and resource sharing play an important role. CP-nets combine the strengths of a high-level programming language. Petri nets provide the primitives for process interaction, while the programming language provides the primitives for the definition of data types and the manipulations of data values. CPNs have an intuitive, graphical representation, which is appealing to human beings (Makajic-Nikolic et al., 2006). A CPN model consists of a set of modules (pages) which each contains a network of places, transitions and arcs. The modules interact with each other through a set of well-defined interfaces, in a similar way as known from many modern programming languages. The graphical representation makes it easy to see the basic structure of a complex CPN model, i.e., understand how the individual processes interact with each other (Jensen K., 1997). It is shown that CPN can handle concepts of dependencies and coordination mechanism as well as by use of decomposition and specialization concepts in an easy way (Makajic-Nikolic D., Vujosevic M., 2002; Malone, T.W., Crowston, K., Lee, J., Pentland, B. et al., 1999).

Numerous papers give examples for the Petri Net usage in modelling and SC analysis. Van der Vorst, et al. (2000) have considered SC as a business process, which is to be redesigned, and in that case many different redesign scenarios are to be tested. PN is used to support a decision-maker in choosing the best-fit scenario. Supply chains in food industry are also modelled in Bhushan N. and K. Gummaraju (2002). The authors propose supply chain management of perishable items combining the TTI and Wireless technologies. They compare the proposed futuristic SC with the existing situation using simulation of Generalized Stochastic PN. An approach to SC Process Management (SCPM) based on high-level Petri-nets, called XML-nets, is presented in von Mevius M. and R. Pibernik (2004). Computer Integrated Manufacturing Open System Architecture (CIMOSA) based process behavior rules and Object-oriented PN (OPTN) are used in Dong M. and F.Frank Chen (2001) to model the routing structures of a typical SC process. In Kemper P. (2000) author show how Generalized Stochastic PN bridges the gap between application formalism like process chains and analysis methods for concurrent systems like PN analysis methods. In Landeghem R.V. and C.-V. Bobeau (2002) the authors propose an implementation of an incremental approach to modeling discrete events systems at the structural level of systems specification. They consider the entire system and coordinate decisions at each stage of the supply chain. Using the example of the Beer Game, a systematic method supports the bottom-up construction of reusable models of supply chains in the Petri nets domain. The paper Makajic-Nikolic D, B. Panić and M. Vujosević (2004) describes a simulation model designed for teaching the bullwhip effect. It explains a simulation model, where all participants behave in the same way, using a timed, hierarchical CPN and software package CPN Tools for the simulation and performance analysis of such a chain.

Supply chain which contains sub suppliers, e.g. suppliers that are hired by the subcontractors, is modeled with PN in (Zegordi and Davarzani, 2012). The authors emphasize the importance of disruptions modeling in the
supply chain, especially when these disorders are caused by inter-state relations. These relations can effect on the relationships between subcontractors, manufacturers and retailers from different countries. PN that models the “quality conflict” between supplier and producer is introduced in the paper (Liu, Fang, Fang and Hipel, 2012). This conflict is caused by the extreme situations that can affect prior made business deals. Methodology which can help the decision maker to develop the PNCA - Petri Net for Conflict Analysis in order to make the right decisions in these conflict situations is presented in this paper.

Zhang and Cheng (2014) propose use of specific class of PN – fuzzy PN for risk modeling and with a possibility to warn of the risk in the supply chain.

**CPN MODEL OF SUPPLY CHAIN**

We have observed simplified SC, with one participant per each phase – a retailer, wholesaler, distributor and manufacturer. The two main activities of each of the participants are:

Delivery of purchased products. At the moment of receiving the order it has to be checked if the inventory stores the needed products. Total amount of the ordered products is delivered if the inventory shows sufficient amounts of products. Otherwise, if the inventory amounts do not suffice the chain participants deliver incomplete orders and form backorder for fulfillment when the next delivery comes.

Forming one’s orders according to new orders. Since a certain amount of products is delivered from a personal inventory, new orders are made from the next SC participant in order to keep the needed level of inventory. We have considered and modeled three different ordering strategies based on three demand forecasting methods: last period demand (LPD), moving average with weighted periods (MAW) and simple exponential smoothing (EXP). If the inventory stores enough products (forecasted trend and safety inventory), the chain participant doesn’t order. If the inventory shows a shortage of products, the amount, that recharges forecasted trend and safety inventory level is ordered.

**Top Page and First Level Sub-pages**

The described SC was modelled by timed Hierarchical Coloured Petri Nets (Jensen K., 1997), and then performed simulation using the software package CPN Tools (http://wiki.daimi.au.dk/cpntools). The net structure consists of one top page and 17 sub-pages in four hierarchical levels, which models decomposition of observed supply chain in sub-processes. The top level of the model is the whole supply chain: a customer and the four mentioned phases, and one sub-page that generate customer’s demands. Figure 2 shows the top-level CPN and demand generating sub-page as well as hierarchical structure of the whole CPN at the left side of the CPN Tools environment.

![Figure 2. Top-level CPN of Simplified Supply Chain](image-url)
A customer is presented by a place named customer whose initial marking represents demand in time initiate. The four phases (participants), a retailer, wholesaler, distributor and manufacturer are presented by means of substitution transitions. A sub-page (i.e. sub-nets: Retailer, Wholesaler, Distributor and Manufacturer) models each participant. The processes modeled by those sub-pages are the specializations described delivery of purchased products and forming one’s orders according to new orders processes. Figure 3 shows sub-pages Retailer and Manufacturer.

CPNs, which model Retailer, Wholesaler, Distributor (i.e. suppliers) are the same. However, a manufacturer is the last SC link and, in a way, he acts in the same manner as suppliers, with the difference that instead of making an order for the next chain participant, he decides what amounts to produce in the following period. Sub-pages for suppliers and manufacturer are detailed explained in Makajić-Nikolić D, B. Panić and M. Vukošević (2004).

![Figure 3. Retailer and Manufacturer Sub-pages](image)

**Demand Forecasting Strategies Sub-pages**

In CPN Simulator we have used three strategies for demand forecasting. First strategy is based just on the past demand. Second and third are based on adaptive time series method: Moving Average and Exponential Smoothing Methods (Chopra S. and P. Meindl, 2001). Hereafter, CPN models of those methods will be explained, as well as the choice one of them during experiments.

Each sub-page at first level has one substitution transition of type strategy_choice (see sub-pages in Figure 3). These substitution transitions are presented by sub-pages type strategy_choice which model choice of forecasting methods for each SC participant. The top sub-page on Figure 4 shows retailer’s sub-page strategy_choice.

We have designed a CPN, which allows each SC participant to choose one of three mentioned strategies. Sub-page strategy choice models this choice using parameters in Declarations at the left side of CPN Tools environment in Figure 4, and guard functions.
The last period demand method is presented by means of transition strategy_latestp on sub-page strategy choice and with guard function [strategy_R=1], which means: this transition may occur only when the value strategy_R in declaration is set to 1. Declaration also shows that the value strategy_R=2 refers to choosing of Weighted Moving Average method, and strategy_R=3 means that Exponential Smoothing method is choisen. R in the name of this parameter refers to the Retailer. Other SC participant has analogous parameters for strategy choice: strategy_W for a wholesaler, strategy_D for a distributor and strategy_M for a manufacturer. Those two strategies are presented by means of sub-pages.

Figure 4 also shows sub-page moving average (at the bottom-right side). In Weighted Moving Average Method we estimate the level of trend in period \( t \) as the average demand over the most recent \( N \) periods. We assign weights \( W_j \) to each period. We assign weights \( W_j \) to each period. This represents a \( N \)-period moving average. Thus, we have the following:

\[
L_t = (p_{t-1}*w_1 + p_{t-2}*w_2 + ... + p_{t-N+1}*w_N)/(w_1 + w_2 + ... + w_{N+1})
\]  

After observing the demand for period \( t+1 \), we revise the estimates as follows:

\[
L_{t+1} = (p_{t+1}*w_{t+1} + p_{t+2}*w_{t+2} + ... + p_{t-N+1}*w_{t-N+1})/(w_{t+1} + w_{t+2} + ... + w_{t-N+2})
\]

In sub-pages type strategy_maw function mav is assigned to the transition moving_average. This function is defined in declaration shown at the left side of the figure 4.

\[
\text{fun mav}(p1,p2,p3,w1,w2,w3)=(p1*w1+p2*w2+p3*w3)\div(w1+w2+w3)
\]

We use 3 last periods for calculate average, and it is possible to define weights for those 3 periods (see declaration in Figure 4).

At the bottom-left side of the Figure 4 CPN model of the Exponential Smoothing Method is shown. In this method the initial estimate \( p_i \) of demand is taken to be the average of all historical data. Demand in the next period is demand forecast for last period \( p_i \) and part of error in the last period \( p_i - p \):

\[
p_{t+1} = p + \alpha(p_t - p)
\]

where \( \alpha \) is a smoothing constant, \( 0 \leq \alpha \leq 1 \).
In sub-pages type strategy_exp, function exp is assigned to the transition strategy_exp. It is defined in declaration shown at the left side of the figure 4.

\[ \text{fun exp(p,alfa,p1)=p+(alfa * (p1-p)) div 10} \] (5)

(This modification of this function is done because of the fact that CPN Tools deal only with integer numbers.)

**Inventory Replenishment Policies Modeling**

After using one of the three strategies, the trend is forecasted. Then, in accordance with this trend, inventory level and function order(i,j) (6), each SC participant calculate one’s order according to new orders.

\[ \text{fun order(i,j)= if (j<i+ls) then (i+ls-j) else 0} \] (6)

Variables i and j represent received order and the inventory, respectively. Function order(i,j) behaves as explained at the beginning of this section. In order experiment with different inventory replenishment policies, level of safety inventory (ls) is given as a parameter.

**SIMULATION RESULTS**

CPN we have formed is the simulator, which allows easy experimenting with four different parameters:

Demand forecasting methods. Each player can choose one of three predefined and modeled demand-forecasting methods and define their parameters and weights. Trend of orders is calculated using these methods.

Inventory control policy. Our CPN simulator allows defining different delivery function. It also allows defining of different behavior CS participants in forming one’s orders according to new orders, calculated trend and the needed level of inventory.

Customer’s demands in time.

The times needed for creating personal orders, the production and delivery procedures. Those times can be deterministic or stochastic.

CPN Tools enables saving of simulation report in the files which can be opened using MS Excel. This report involves all data about simulation flow, i.e. transition firings: simulations step, time and used tokens. However, from such a format of report it is very difficult to extract and analyze the data. This is why we use MS Excel macros for the transformation and analysis of simulation reports. According to the simulations results we can measure different performances whose changes are the consequences of player’s decisions: reactions of participants to sudden changes in customer demands (the bullwhip effect), changes in inventory in a time, surplus of inventory goods, changes in inventory costs and backorder costs in a time, inventory level at the end of simulated period, total costs of each SC participants and of the entire SC.

**Experimental Results**

Using CPN simulator we have made three groups of experiments. In each group we observe the four mentioned parameters (demand forecasting methods, inventory control policy, customer’s demands and the lead times) and vary one of them. All simulations are replicated 30 times, and then the average values are calculated.

**I group of experiments – four different parameters for Exponential Smoothing Method**

Demand forecasting method: Exponential Smoothing Method, \( \alpha = \{0.1, 0.2, 0.3, 0.4\} \).

Inventory control policy. fun order (i,j)= if (j<i+60) then (i+60-j) else 0.

Customer’s demands. For first 12 weeks demand is: 60, 60, 60, 60, 120, 150, 180, 170, 180, 150, 200, 170. For the rest of the year (40 weeks) customer’s demands are generated using function \( \text{fun Norm} \) (120,10) and sub-net demand generating.

The lead times: 2 weeks
Results:

Table 1. SC Costs for four different parameters for Exponential Smoothing Method

<table>
<thead>
<tr>
<th>α</th>
<th>Average SC inventory costs</th>
<th>Average SC backorder costs</th>
<th>Average SC total costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>8356.6</td>
<td>10179.3</td>
<td>18536.0</td>
</tr>
<tr>
<td>0.2</td>
<td>12754.0</td>
<td>9917.4</td>
<td>22671.4</td>
</tr>
<tr>
<td>0.3</td>
<td>13728.3</td>
<td>9973.3</td>
<td>23701.6</td>
</tr>
<tr>
<td>0.4</td>
<td>14144.5</td>
<td>9920.4</td>
<td>24064.8</td>
</tr>
</tbody>
</table>

The table 2 shows that the costs of entire supply chain are minimal for \( \alpha = 0.1 \). That is the reason way the rest of the experiments are made for \( \alpha = 0.1 \) when the Exponential Smoothing Method is used.

II group of experiments – different demand forecasting methods

Demand forecasting methods:
The last period demand method (LP)
Weighted Moving Average Method (MAW) for 3 periods and weights: \( w_1 = 0.1, w_2 = 0.3, w_3 = 0.6 \)
Exponential Smoothing Method (EXP), \( \alpha=0.1 \)
Inventory control policy. fun order \((i,j)=\) if \((j<i+60)\) then \((i+60-j)\) else 0.
Customer’s demands. For first 12 weeks demand is: 60, 60, 60, 60, 120, 150, 180, 170, 180, 150, 200, 170. For the rest of the year (40 weeks) customer’s demands are generated using function \(fun\ Norm\) (120,10) and sub-net demand generating.
The lead times: 2 weeks

Results:

Table 2. Average Order’s Peaks for SC Participants

<table>
<thead>
<tr>
<th>Demand</th>
<th>Retailer’s average peak</th>
<th>Wholesaler’s average peak</th>
<th>Distributor’s Average Peak</th>
<th>Manufacturer’s Average peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>354.3</td>
<td>309.0</td>
<td>279.1</td>
<td>239.6</td>
</tr>
<tr>
<td>MAW</td>
<td>309.8</td>
<td>285.7</td>
<td>257.8</td>
<td>232.5</td>
</tr>
<tr>
<td>EXP</td>
<td>250.4</td>
<td>235.4</td>
<td>212.9</td>
<td>182.0</td>
</tr>
</tbody>
</table>

Table 2 and Figure 6 show the expected results: the last period demand method gives the biggest bullwhip effect, and the exponential smoothing method gives the lowest bullwhip effect. However, table 3 shows unexpected result: all methods give the similar costs.
Table 3. Average Inventory And Backorder Costs For SC Participants

<table>
<thead>
<tr>
<th>Average costs</th>
<th>Manufacturer</th>
<th>Distributor</th>
<th>Wholesaler</th>
<th>Retailer</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP Inventory</td>
<td>2230,45</td>
<td>2651,33</td>
<td>2210,92</td>
<td>1023,93</td>
<td>8116,63</td>
</tr>
<tr>
<td>Backorder</td>
<td>2663,07</td>
<td>2894,13</td>
<td>2497,30</td>
<td>2009,77</td>
<td>10064,27</td>
</tr>
<tr>
<td>Total</td>
<td>4893,52</td>
<td>5545,47</td>
<td>4708,22</td>
<td>3033,70</td>
<td>18180,90</td>
</tr>
<tr>
<td>MAW Inventory</td>
<td>2101,82</td>
<td>2894,82</td>
<td>2231,67</td>
<td>1188,35</td>
<td>8416,65</td>
</tr>
<tr>
<td>Backorder</td>
<td>2818,70</td>
<td>2679,50</td>
<td>2503,73</td>
<td>1865,27</td>
<td>9867,20</td>
</tr>
<tr>
<td>Total</td>
<td>4920,517</td>
<td>5574,317</td>
<td>4735,4</td>
<td>3053,617</td>
<td>18283,85</td>
</tr>
<tr>
<td>EXP Inventory</td>
<td>1689,85</td>
<td>2886,77</td>
<td>2519,03</td>
<td>1260,97</td>
<td>8356,62</td>
</tr>
<tr>
<td>Backorder</td>
<td>3165,83</td>
<td>2618,43</td>
<td>2595,77</td>
<td>1799,30</td>
<td>10179,33</td>
</tr>
<tr>
<td>Total</td>
<td>4855,68</td>
<td>5505,20</td>
<td>5114,80</td>
<td>3060,27</td>
<td>18535,95</td>
</tr>
</tbody>
</table>

III group of experiments – different lead time

Demand forecasting method: Exponential Smoothing Method (EXP), $\alpha=0.1$

Inventory control policy. fun order ($i,j$) = if ($j < i+60$) then ($i+60-j$) else 0.

Customer’s demands. For first 12 weeks demand is: 60, 60, 60, 60, 120, 150, 180, 170, 180, 150, 200, 170. For the rest of the year (40 weeks) customer’s demands are generated using function $\text{fun Norm}(120,10)$ and sub-net demand generating.

The lead times: 2 weeks and 1 week

Results

Table 4. Average Inventory and Backorder Costs

<table>
<thead>
<tr>
<th>Average costs</th>
<th>Manufacturer</th>
<th>Distributor</th>
<th>Wholesaler</th>
<th>Retailer</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead time 2</td>
<td>1689,85</td>
<td>2886,77</td>
<td>2519,03</td>
<td>1260,97</td>
<td>8356,62</td>
</tr>
<tr>
<td>Backorder</td>
<td>3165,83</td>
<td>2618,43</td>
<td>2595,77</td>
<td>1799,30</td>
<td>10179,33</td>
</tr>
<tr>
<td>Total</td>
<td>4855,68</td>
<td>5505,20</td>
<td>5114,80</td>
<td>3060,27</td>
<td>18535,95</td>
</tr>
<tr>
<td>Lead time 1</td>
<td>1450,42</td>
<td>1346,60</td>
<td>1199,72</td>
<td>896,70</td>
<td>4893,43</td>
</tr>
<tr>
<td>Backorder</td>
<td>2114,33</td>
<td>1897,50</td>
<td>2060,33</td>
<td>1606,90</td>
<td>7679,07</td>
</tr>
<tr>
<td>Total</td>
<td>3564,75</td>
<td>3244,10</td>
<td>3260,05</td>
<td>2503,60</td>
<td>12572,50</td>
</tr>
</tbody>
</table>

Table 4 and Figure 7 show the expected results: longer lead time results in higher costs and bigger bullwhip effect.

Figure 7. Bullwhip Effects For Different Lead Times

CONCLUDING REMARKS

The paper describes how we have modeled CPN simulator of order and delivery processes of the purchased amounts processes in a supply chain. We have designed simulator that allows one participant to simultaneously experiment with the parameters of all SC stages. Furthermore, using this simulator, a player can define in advance the rules of demand forecasting and inventory control policy. According to the results of simulations we can measure different performances whose changes are the consequences of player’s decisions: reactions of participants to sudden changes in customer demands (the bullwhip effect), changes in supply level in a time, surplus of inventory goods, the costs of SC participants and of the entire SC, etc. This simulator is developed using hierarchical Coloured Petri Nets (CPN). We have made three groups of experiments and obtained some expected but also some unexpected results. We have shown that it is possible to form a CPN model of a supply chain, which allows easy experimenting with different parameters and strategies of all SC stages.
REFERENCES


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The Near Beer Game at http://www.forio.com/nearbeer.htm


