ON PSEUDO-SYMMETRY CURVATURE CONDITIONS OF GENERALIZED \((k,\mu)\)-PARACONTACT METRIC MANIFOLDS

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Abstract. In this paper we investigate Ricci pseudo-symmetric and Ricci generalized pseudo-symmetric generalized \((k,\mu)\)-paracontact metric manifolds. Besides this we characterize generalized \((k,\mu)\)-paracontact metric manifolds satisfying the curvature conditions \(Q(S,R) = 0\) and \(Q(S,g) = 0\), where \(S, R\) are the Ricci tensor and curvature tensor respectively. Several corollaries are also obtained.

1. Introduction

The notion of paracontact geometry was introduced by Kaneyuki and Williams [16] in 1985. A systematic investigation on paracontact metric manifolds done by Zamkovoy [19]. Recently, Cappelletti-Montano et al [6] introduced a new type of paracontact geometry so-called paracontact metric \((k,\mu)\) space, where \(k\) and \(\mu\) are constant. It is known [1] that in contact case \(k \leq 1\), but in paracontact case there is no restriction for \(k\).

The conformal curvature tensor \(C\) is invariant under conformal transformation and vanishes identically for 3-dimensional manifolds. Using this result several authors studied different types of 3-dimensional manifolds ([10], [11], [12]).

A semi-Riemannian manifold \((M, g)\) is called locally symmetric if its curvature tensor \(R\) is parallel (that is, \(\nabla R = 0\)) and semi-symmetric if its curvature tensor \(R\) satisfies the condition

\[
R(X,Y) \cdot R = 0,
\]

where \(R\) is the Riemannian curvature tensor and \(R(X,Y)\) is considered as a derivation of the tensor algebra at each point of the manifold for tangent vector fields.
A complete intrinsic classification of these manifolds was given by Szabo in [18].

A \((k,\mu)\)-paracontact metric manifold is called an Einstein manifold if the Ricci tensor satisfies the condition \(S = \lambda g\), where \(\lambda\) is some constant.

We define endomorphisms \(R(X,Y)\) and \(X \wedge A Y\) by

\[
R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z
\]

and

\[
(X \wedge A Y)Z = A(Y,Z)X - A(X,Z)Y,
\]

respectively, where \(X,Y,Z \in \chi(M)\), \(\chi(M)\) is the set of all differentiable vector fields on \(M\), \(A\) is the symmetric \((0,2)\)-tensor, \(R\) is the Riemannian curvature tensor of type \((1,3)\) and \(\nabla\) is the Levi-Civita connection. For a \((0,k)\)-tensor field \(T\), \(k \geq 1\), on \((M,g)\) we define the tensor \(R \cdot T\) and \(Q(g,T)\) by

\[
(R(X,Y) \cdot T)(X_1, X_2, \ldots, X_k) = -T(R(X,Y)X_1, X_2, \ldots, X_k) - T(X_1, R(X,Y)X_2, \ldots, X_k) - T(X_1, X_2, \ldots, R(X,Y)X_k)
\]

\[
Q(g,T)(X_1, X_2, \ldots, X_k, Y) = -T((X \wedge Y)X_1, X_2, \ldots, X_k) - T(X_1, (X \wedge Y)X_2, \ldots, X_k) - T(X_1, X_2, \ldots, (X \wedge Y)X_k)
\]

respectively [17]. If the tensors \(R \cdot S\) and \(Q(g,S)\) are linearly dependent, then \(M\) is called Ricci pseudo-symmetric [17]. This is equivalent to

\[
R \cdot S = fQ(g,S),
\]

holding on the set \(U_S = \{x \in M : S \neq 0 \text{ at } x\}\), where \(f\) is some function on \(U_S\). Also if the tensors \(R \cdot R\) and \(Q(S,R)\) are linearly dependent, then \(M\) is said to be Ricci generalized pseudo-symmetric [17]. This is equivalent to

\[
R \cdot R = fQ(S,R).
\]

Recently, 3-dimensional generalized \((k,\mu)\)-paracontact metric manifolds have been studied by Kupeli Erken et al ([15], [14]). Kowalczyk [13] studied semi-Riemannian manifolds satisfying \(Q(S,R) = 0\) and \(Q(g,S) = 0\), where \(S, R\) are the Ricci tensor and curvature tensor respectively. De et al. [9] studied Ricci pseudo-symmetric and Ricci generalized pseudo-symmetric \(P\)-sasakian manifolds.

The paper is organized in the following way:

In Section 2, we discuss about some basic results of paracontact metric manifolds. Next, we investigate Ricci pseudo-symmetric generalized \((k,\mu)\)-paracontact metric manifolds. Section 4 deals with Ricci generalized pseudo-symmetric generalized \((k,\mu)\)-paracontact metric manifolds. In Section 5 and 6 we study generalized \((k,\mu)\)-paracontact metric manifolds satisfying \(Q(S,R) = 0\) and \(Q(S,g) = 0\), where \(S, R\) are the Ricci tensor and curvature tensor respectively.
2. Preliminaries

A (2n+1)-dimensional smooth manifold $M$ is said to be has an almost paracon-tact structure if it carries a (1,1)-tensor $\phi$, a vector field $\xi$ and a 1-form $\eta$ satisfying [16]:

(i) $\phi^2 X = X - \eta(X)\xi$, for all $X \in \chi(M)$, $\eta(\xi) = 1$,

(ii) the tensor field $\phi$ induces an almost paracomplex structure on each fibre of $D = \ker(\eta)$, that is, the eigendistributions $D^+_{\phi}$ and $D^-_{\phi}$ of $\phi$ corresponding the eigenvalues 1 and -1, respectively, have equal dimension $n$.

From the above conditions it follows that $\phi(\xi) = 0$, $\eta \circ \phi = 0$.

An almost paraccontact structure is said to be normal [16] if and only if the (1,2) type torsion tensor $N_{\phi} = [\phi, \phi] - 2d\eta \otimes \xi$ vanishes identically, where $[\phi, \phi](X,Y) = \phi^2[X,Y] + [\phi X, \phi Y] - \phi[\phi X, Y] - \phi[X, \phi Y]$. If an almost paraccontact manifold admits a pseudo-Riemannian metric $g$ such that

$g(\phi X, \phi Y) = -g(X, Y) + \eta(X)\eta(Y)$,

for $X, Y \in \chi(M)$, then we say that $(M, \phi, \xi, \eta, g)$ is an almost paraccontact metric manifold. Any such pseudo-Riemannian metric manifold is of signature $(n+1, n)$. An almost paraccontact structure is said to be a paraccontact structure if $g(\xi, \phi Y) = d\eta(\xi, Y)$ [19]. In a paraccontact metric manifold we define (1,1)-type tensor fields $h$ by $h = 1/2 \mathcal{L}_\xi \phi$, where $\mathcal{L}_\xi \phi$ is the Lie derivative of $\phi$ along the vector field $\xi$. Then we observe that $h$ is symmetric and anti-commutes with $\phi$. Also $h$ satisfies the following conditions [19]:

$\nabla_X \xi = -\phi X + \phi h X$.

Moreover $h$ vanishes identically if and only if $\xi$ is a Killing vector field and then $(M, \phi, \xi, \eta, g)$ is said to be a $K$-paraccontact manifold. $(k, \mu)$-paraccontact manifolds have been studied by Calvasuso et al. ([3],[4], [5]) and Cappellaet-Montano et al. ([7], [8]) and many others.

Generalized $(k, \mu)$-paraccontact metric manifolds were studied by Murathan and Kupeli Erken in [15]. A generalized $(k, \mu)$-paraccontact metric manifolds mean a 3-dimensional paraccontact metric manifold which satisfy the nullity condition

$R(X,Y)\xi = k(\eta(Y)X - \eta(X)Y) + \mu(\eta(Y)hX - \eta(X)hY)$.

In a generalized $(k \neq -1, \mu)$-paraccontact manifold the following results hold ([2], [14]):

$R(X,Y)\xi = k(\eta(Y)X - \eta(X)Y) + \mu(\eta(Y)hX - \eta(X)hY)$.

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for all $X \in \chi(M)$, where $\nabla$ denotes the Levi-Civita connection of the pseudo-Riemannian manifold.

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Lemma 2.1. [14] Let $M(\phi, \xi, \eta, g)$ be a generalized $(k, \mu)$-paracontact metric manifold with $k > -1$ and $\xi \mu = 0$. Then

1. At any point of $M$, precisely one of the following relations is valid: $\mu = 2(1 + \sqrt{1 + k})$, or $\mu = 2(1 - \sqrt{1 + k})$

2. At any point $P \in M$ there exists a chart $(U, (x, y, z))$ with $P \in U \subseteq M$, such that the functions $k, \mu$ depend only on the variable $z$.

3. Ricci pseudo-symmetric generalized $(k, \mu)$-paracontact metric manifolds

In this section we study Ricci pseudo-symmetric generalized $(k, \mu)$-paracontact metric manifolds, that is, the manifold satisfying the curvature condition $R \cdot S = fQ(g, S)$. Then we have from (1.6)

\[(R(X, Y) \cdot S)(U, V) = fQ(g, S)(X, Y; U, V).\]

It is equivalent to

\[(R(X, Y) \cdot S)(U, V) = f((X \wedge g Y) \cdot S)(U, V)).\]

Using (1.7) in (3.2), we get

\[-S(R(X, Y)U, V) - S(U, R(X, Y)V) = f[-g(Y, U)S(X, V) + g(X, U)S(Y, V) - g(Y, V)S(U, X) + g(X, V)S(U, Y)].\]

Substituting $X = U = \xi$, we obtain

\[-S(R(\xi, Y)\xi, V) - S(\xi, R(\xi, Y)V) = f[-g(\xi, \xi)S(\xi, V) + g(\xi, \xi)S(Y, V) - g(\xi, V)S(\xi, \xi) + g(\xi, V)S(\xi, Y)].\]

Applying (2.4) and (2.7) in (3.4), we get

\[(k - f)[S(Y, V) - 2kg(Y, V)] + \mu[S(hY, V) - 2kg(hY, V)] = 0.\]

Putting $hY$ for $Y$ in (3.5) yields

\[(k - f)[S(hY, V) - 2kg(hY, V)] + \mu(k + 1)[S(Y, V) - 2kg(Y, V)] = 0.\]

Multiplying (3.5) by $(k - f)$ and (3.6) by $\mu$ and subtracting the results we have

\[(k - f)^2 - \mu^2(k + 1)[S(Y, V) - 2kg(Y, V)] = 0.\]

Then either $S(Y, V) = 2kg(Y, V)$ or, $(k - f)^2 = \mu^2(k + 1)$.

Case 1: Let $S(Y, V) = 2kg(Y, V)$. Then the manifold is an Einstein manifold.

Case 2: Let $(k - f)^2 = \mu^2(k + 1)$. Therefore $f = k \pm \mu \sqrt{1 + k}$. Hence the manifold is of the form $R \cdot S = (k \pm \mu \sqrt{1 + k})Q(g, S)$.

By the above discussions we have the following:

Theorem 3.1. A Ricci pseudo-symmetric generalized $(k, \mu)$-paracontact metric manifold is either an Einstein manifold or of the form $R \cdot S = (k \pm \mu \sqrt{1 + k})Q(g, S)$.

Also we can state the following:
Proposition 3.1. Every Ricci pseudo-symmetric generalized \((k, \mu)\)-paracontact metric manifold is of the form \(R \cdot S = (k \pm \mu \sqrt{1+k})Q(g, S)\), provided the manifold is non-Einstein.

If the manifold is an Einstein manifold, then obviously the manifold is Ricci pseudo-symmetric. This leads to the following:

Corollary 3.1. A generalized \((k, \mu)\)-paracontact metric manifold is Ricci pseudo-symmetric if and only if the manifold is an Einstein manifold, provided \(f \neq k \pm \mu \sqrt{1+k}\).

4. Ricci Generalized Pseudo-Symmetric Generalized \((k, \mu)\)-Paracontact Metric Manifolds

This section is devoted to study Ricci generalized pseudo-symmetric generalized \((k, \mu)\)-paracontact metric manifolds. Then we have \(R \cdot R = fQ(S, R)\), that is,

\[(R(X, Y) \cdot R)(U, V)W = f((X \wedge_S Y) \cdot R)(U, V)W.\]

Then using (1.6) in (4.1), we get

\[
\]

Putting \(X = U = \xi\) in (4.2), we have

\[
R(\xi, \xi)R(\xi, \xi)W - R(R(\xi, Y)\xi, V)W - R(\xi, R(\xi, Y)V)W \\
- R(\xi, V)R(\xi, Y)V = f[S(Y, R(\xi, Y)V)\xi - S(\xi, R(\xi, Y)V)Y \\
- S(Y, \xi)R(\xi, V)W + S(\xi, \xi)R(Y, V)W - S(Y, V)R(\xi, \xi)V \\
+ S(\xi, \xi)R(\xi, \xi)W - S(Y, W)R(\xi, \xi)V + S(\xi, W)R(\xi, \xi)V].
\]

Applying (2.4) and (2.7) in (4.3), we get

\[
-k^2g(V, W)Y - \mu kg(V, W)hY - \mu^2g(Y, W)hV + \mu^3g(W, V)hY \\
- \mu kg(hW, V)Y - \mu^2g(hW, V)hY + \mu^3g(W, hV)hY \\
+ kR(Y, V)W + \mu R(hY, V)W + \mu kg(hY, W)\eta(V)\xi - \\
\mu^2g(W, V)hV + \mu^3g(hW, V)hV + kR(Y, V)W \\
- \mu^2g(hW, Y)hV = f[-k\eta(W)S(Y, V)\xi - k\eta(W)S(Y, hV)\xi \\
- \mu^2g(Y, W)hV - 2k\eta(W)S(Y, hV)hV \\
+ 2k^2\eta(W)g(Y, W)\xi + 2k^2\eta(W)g(Y, hV)hV \\
- 2k\eta(W)S(Y, hV)hV + S(W, hV)hV + \mu S(Y, W)hV + 2k^2\eta(W)g(Y, Y)\xi \\
+ 2k\eta(W)g(hY, V)\xi].
\]
Taking inner product with $T$, we obtain
\begin{align*}
-k^2g(V,W)g(Y,T) - \mu kg(V,W)g(hY,T) - \mu \kappa g(W)g(hY,Y)\eta(T)
-\mu kg(hW,V)g(Y,T) - \mu^2g(hW,V)g(hY,T) + \mu \kappa g(W)g(Y,hV)\eta(T)
+ kg(R(Y,V)W,T) + \mu g(R(hY,V)W,T) + \mu kg(hY,V)\eta(V)\eta(T)
- \mu \kappa g(V)\eta(W)g(hY,T) + \mu^2(k+1)\eta(V)g(Y,W)\eta(T)
- \mu^2(k+1)\eta(V)\eta(W)g(Y,T) + k^2g(Y,W)g(V,T)
+ \mu kg(Y,W)g(hV,T) + + \mu kg(hW,Y)g(V,T) + \mu^2g(hW,Y)g(hV,T).
\end{align*}

(4.5) $+ 2k\mu \eta(W)g(hY,V)\eta(T)$.

Let $\{e_i\}, i = 1, 2, 3$ be a local orthonormal basis in the tangent space $T_p M$ at each point $p \in M$. Substituting $Y = T = e_i$ in (4.5) and summing over $i = 1$ to 3, we infer that

(4.6) $(1-3f)k\{S(Y,T) - 2kg(Y,T)\} + \mu(1-f)\{S(hY,T) - 2kg(hY,T)\} = 0$.

Setting $hY$ for $Y$ in (4.6), we get

(4.7) $(1-3f)k\{S(hY,T) - 2kg(hY,T)\} + \mu(1-f)(k+1)\{S(Y,T) - 2kg(Y,T)\} = 0$.

Multiplying (4.6) by $(1-3f)k$ and (4.7) by $\mu(1-f)$ and then subtracting the result, we have

(4.8) $\{(1-3f)^2k^2 - \mu^2(1-f)^2(k+1)\}\{S(Y,T) - 2kg(Y,T)\} = 0$.

Then either $S(Y,T) = 2kg(Y,T)$

or, $(1-3f)^2k^2 - \mu^2(1-f)^2(k+1) = 0$.

Thus we can state the following:

**Theorem 4.1.** A Ricci generalized pseudo-symmetric generalized $(k,\mu)$-paracontact metric manifold is an Einstein manifold, provided $(1-3f)^2k^2 - \mu^2(1-f)^2(k+1) \neq 0$.

Now if we consider $\mu = 0$, then from $(1-3f)^2k^2 - \mu^2(1-f)^2(k+1) = 0$, we infer $f = \frac{1}{3}$. Thus we can state that

**Corollary 4.1.** A Ricci generalized pseudo-symmetric generalized $N(k)$-paracontact metric manifold is of the form $R \cdot R = \frac{1}{4}Q(S,R)$, provided the manifold is non-Einstein.

Again if we consider $f = 0$, then from $(1-3f)^2k^2 - \mu^2(1-f)^2(k+1) = 0$, we obtain

(4.9) $k^2 - \mu^2(k+1) = 0$,

which implies $(2k - \mu^2)(\xi k) - 2\mu(k+1)(\xi \mu) = 0$. Now by using (2.6) we have $\mu(k+1)(\xi \mu) = 0$. Taking account of $\mu \neq 0$ and $k < -1$, we have $\xi \mu = 0$. Hence using Lemma 2.1 we have the following:
Corollary 4.2. If a generalized \((k, \mu)\)-paracontact metric manifold with \(k > -1\) satisfy the curvature condition \(R \cdot R = 0\) then at any point \(P \in M\) there exists a chart \((U, (x, y, z))\) with \(P \in U \subseteq M\), such that the functions \(k, \mu\) depend only on the variable \(z\) and either \(\mu = 2(1 + \sqrt{1 + k})\), or \(\mu = 2(1 - \sqrt{1 + k})\) is valid.

5. GENERALIZED \((k, \mu)\)-PARACONTACT METRIC MANIFOLD SATISFYING \(Q(S,R)=0\)

In this section we study generalized \((k, \mu)\)-paracontact metric manifolds satisfying the curvature condition \(Q(S,R)=0\). Therefore

\[
(X \wedge S Y) \cdot R(U, V)W = 0.
\]

Then using (1.7) in (5.1), we get

\[
\]

(5.2)

\]

Substituting \(X = U = \xi\) in (5.2) yields

\[
S(Y, R(\xi, V)W)\xi - S(\xi, R(\xi, V)W)Y - S(Y, \xi)R(\xi, V)W
+ S(\xi, \xi)R(Y, V)W - S(Y, V)R(\xi, \xi)W + S(\xi, V)R(\xi, Y)W
\]

(5.3)

\[-S(Y, W)R(\xi, V)\xi + S(\xi, W)R(\xi, V)Y = 0.
\]

Applying (2.4) and (2.7) in (5.3), we get

\[
-k\eta(W)S(Y, V)\xi - \mu\eta(W)S(Y, hV)\xi - 2k^2g(V, W)Y - 2gW(Y, V)Y
+ 2kR(Y, V)W + 2k^2\eta(V)g(Y, W)\xi + 2\mu g(hW, Y)\eta(V)J - 2\mu g(\eta(V))\eta(W)hY
\]

\[-k\eta(V)S(Y, W)\xi + kS(Y, W)V + \mu S(Y, W)hV + 2k^2\eta(W)g(V, Y)\xi
\]

(5.4)

\[+ 2k\mu g(W)g(hY, V)\xi = 0.
\]

Taking inner product with \(T\), we obtain

\[
-k\eta(W)S(Y, V)\eta(T) - \mu\eta(W)S(Y, hV)\eta(T) - 2k^2g(V, W)Y
- 2gW(Y, V)\eta(T) + 2kR(Y, V)W + 2k^2\eta(V)g(Y, W)\eta(T)
+ 2\mu g(hW, Y)\eta(V)\eta(T) - 2\mu g(\eta(V))\eta(W)g(hY, T) - k\eta(V)S(Y, W)\eta(T)
\]

\[+ kS(Y, W)g(hV, T) + \mu S(Y, W)g(hV, T) + 2k^2\eta(W)g(V, Y)\eta(T)
\]

(5.5)

\[+ 2k\mu g(W)g(hY, V)\eta(T) = 0.
\]

Let \(\{\epsilon_i\}, i = 1, 2, 3\) be a local orthonormal basis in the tangent space \(T_p M\) at each point \(p \in M\). Substituting \(Y = T = \epsilon_i\) in (5.5) and summing over \(i = 1\) to 3, we have

\[
-6k^2g(Y, T) + 3kS(Y, T) - 2k\mu g(hY, T) + \mu S(hY, T) = 0.
\]

Putting \(Y = hY\) in (5.6), we get

\[
-6k^2g(hY, T) + 3kS(hY, T) - 2(k + 1)k\mu g(Y, T) + \mu (k + 1)S(Y, T) = 0.
\]

Multiplying (5.6) by \(3k\) and (5.7) by \(\mu\) and then subtracting the result we have

\[
(9k^2 - \mu^2(k + 1))(S(Y, T) - 2kg(Y, T)) = 0.
\]

Then either \(9k^2 - \mu^2(k + 1) = 0\) or, \(S(Y, T) = 2kg(Y, T)\).

Thus we can state the following:
Theorem 5.1. If a generalized $(k,\mu)$-paracontact metric manifold satisfy the condition $Q(S,R) = 0$, then the manifold is an Einstein manifold, provided $9k^2 - \mu^2(k+1) \neq 0$

6. GENERALIZED $(k,\mu)$-PARACONTACT METRIC MANIFOLDS SATISFYING $Q(g,S)=0$

In this section we investigate generalized $(k,\mu)$-paracontact metric manifolds satisfying $Q(g,S) = 0$. Therefore

\begin{equation}
(X \wedge g Y \cdot S)(U,V) = 0
\end{equation}

Using (1.6) in (6.1), we get

\begin{equation}
-g(Y,U)S(X,V) + g(X,U)S(Y,V) - g(Y,V)S(U,X) + g(X,V)S(U,Y) = 0.
\end{equation}

Substituting $X = U = \xi$, we obtain

\begin{equation}
-g(Y,\xi)S(\xi,V) + g(\xi,\xi)S(Y,V) - g(Y,V)S(\xi,\xi) + g(\xi,V)S(\xi,Y) = 0.
\end{equation}

Applying (2.4) and (2.7) in (6.3), we get

\begin{equation}
S(Y,V) - 2kg(Y,V) = 0.
\end{equation}

This leads to the following:

Theorem 6.1. If a generalized $(k,\mu)$-paracontact metric manifold satisfy the condition $Q(g,S) = 0$, then the manifold is an Einstein manifold.

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