Causal Effects and Dynamic Relationship between Exchange Rate Volatility and Economic Development in Liberia

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ABSTRACT

Studies envisioned to inform on major policy issues are paramount for Liberia’s economic recovery. Therefore, we employ an unrestricted vector autoregressive model to analyze the dynamic associations between exchange rate volatility (ERV) and Liberia’s real gross domestic product (RGDP). The empirical results show no significant relationship between ERV and Liberia’s RGDP in the short-run, but variance decomposition analysis reveals that innovations to Liberia’s RGDP lead to fluctuations in ERV in the long-run. Hence, we recommend that Liberia’s policymakers should exert stronger monetary policy control to ensure the existence of single currency regime in the long-run. Also, technological innovation is required to boost domestic production in order to offset the negative effect of ERV on trade.

Keywords: Economic Growth, Exchange Rate Volatility, Liberia, Unrestricted Vector Autoregressive Model

JEL Classifications: E31, E32, E52

1. INTRODUCTION

The high degree of exchange rate (EXC) fluctuation and its leverage effects on economic activities have raised serious concern among macroeconomists and policymakers. Exchange rate volatility (ERV), as defined by Ozturk (2006), is a statistical measure of the propensity of EXC to increase or decrease within a period. According to Marston (1988), it is linked with currency appreciation and depreciation. On this note, reference is made to two schools of thoughts. On the one hand, Gupta-Kapoor and Ramakrishnan (1999) demonstrate that currency depreciation leads to enhancement of trade balance (TB) in the long-term. On the other hand, Bahmani-Oskooee and Alse (1994) argue that depreciation of EXC has an adverse impact on TB. However, Samuelson and Nordhaus (2001) show that EXCs are mainly unstable in the short-run because they are very sensitive to intervention policies of central banks, monetary policy, changes in expectations, etc. and in the long-term they are affected by relative commodity prices. Therefore, knowledge of ERV is important in understanding the foreign exchange market behavior and how different EXC regimes affect the level of economic activities. This is because uncertainty about EXC movements causes uncertainty in investment decisions, formulation of macroeconomic policy, international trade flows and can even influence the general level of prices.

Consequently, researchers have employed several methods to scrutinize EXC behavior. Some studies use volatility, variance, standard deviation, and range. For example, Bleaney and Francisco (2008) argue that higher range and standard deviation are indicative of higher volatility behavior. However, since the onset of the autoregressive conditional heteroskedastic (ARCH) and general autoregressive conditional heteroskedastic (GARCH) models developed by Engle (1982) and Bollerslev (1986), methods of gauging volatility have transcended from just using range and standard deviation. For instance, ERV has been modeled in the past three decades by application of parametric (variants of GARCH modeling) and non-parametric techniques such as bi-power and truncated power variations, historical and realized volatilities among others (Erdemlioglu et al., 2012). However, econometric theory has not clearly provided an authoritative method of computing volatility (Adeoye and Atanda, 2011). Therefore, to accomplish the purpose of this research, we compute the annual changes (volatility) in official EXCs by taking the
natural logarithmic difference between two annual EXC values. Then, we employ an unrestricted vector autoregressive (U-VAR) model to explore leverage effects of ERV on key macroeconomic variables in Liberia.

From an economic point of view, ERV affects the level of economic development as envisaged by the way it influences domestic prices and international trade. This is because a rise in EXC uncertainty has income and substitution effects. On the one hand, the income effect may raise trade activity, because higher ERV expectation presents greater opportunity for profit-seeking and might increase trade. On the other hand, the substitution effect causes merchants to divert from foreign trade towards domestic trade. On this note, it is debated that an increase in the level of ERV raises the uncertainty of profits on transactions denominated in a foreign currency. Consequently, it has the propensity to redirect investment activity away from highly volatile foreign market environments to the lower risk domestic market economy. With this in mind, it becomes important to study the EXC – macroeconomic nexus in Liberia.

Let us not forget that the domestic market in Liberia is a dollarized market economy, having parallel dual currency – the Liberian dollars (LRD) and the United States dollars (USD). In Liberia, one can directly trade in both currencies in the local market. With the USD being a leading international currency, appreciation of the USD exerts pressure on the LRD, thereby raising the general level of prices and inducing inflation in the Liberian economy. Meanwhile, Liberia has a long standing history of using different currencies as legal tenders as reported in Table 1.

Like any developing country, Liberians depend on remittances from the rest of the world. Moreover, the lack of developed manufacturing sector compels traders to import foreign goods, thereby making Liberia an import dependent nation. Since importation of foreign goods is done with international currencies, the demand for foreign currencies is high in Liberia, thereby creating volatile EXC regime. Therefore, there is no doubt that Liberia’s macro-economy is susceptible to any adverse changes in EXC regime.

As can be seen from Figure 1, the EXC regime in Liberia has been very volatile in recent years. Therefore, it is very important to gauge the EXC – macroeconomic behavior in order to advise on key policy issues. Based on the volatile nature of EXC movement in Liberia and the associated uncertainty, the objective of this paper is to utilize a more recent dataset for the purpose of examining the associations between ERV and key macroeconomic variables in Liberia, with the courage of detecting whether ERV has leverage effects on economic development, and thus allowing for a richer insight on the EXC thesis within the framework of Liberia. In order to realize the objective of the study, an unrestricted VAR model shall be applied to access whether ERV has leverage effects on key macroeconomic activities in Liberia. If so, then we shall find out its extent. Also, the study intends to explode the broader implications and bearings of ERV on economic developments in Liberia.

Indeed, the scientific contributions and originality and of this paper add value to the literature because this study offers the first-of-its-kind empirical approach to the evaluation of EXC behavior in Liberia. Thus, the writers believe that the selected variables and study period are sufficient to generalize the results to other developing countries. Nevertheless, let us quickly note that the lack of data has sometimes been one of the main reasons for less empirical studies in Liberia and a major disadvantage for their validity. While the results articulated in this study have broader implications, particularly for developing nations, the attention on Liberia is based by numerous factors. To begin with, war and civil strife in Liberia have eroded much of its economy and infrastructures, making it challenging to reach acceptable level of development. This is one reason why Liberia currently lacks a structured financial market and advanced monetary policy. Consequently, that floating EXC regime may have negative effects on economic development in Liberia. Additionally, as Liberia pursues for economic growth and development, the agendas for poverty alleviation and economic growth demand a comprehensive insight of EXC performance. This study, therefore, attempts to bring to the front Liberia’s macroeconomic needs and offers significant opportunities for Liberia’s monetary policy frameworks.

The rest of the paper is structured as follows: Section 2 summarizes the relevant empirical literatures. In Section 3, the research variables and data are discussed, while Section 4 outlines the methods and estimation procedures. In Section 5, the key empirical results are given and discussed. Finally, Section 6 provides concluding remarks and key policy measures.

### 2. LITERATURE REVIEW

Empirical studies on the interactions between ERV and key macroeconomic variables are voluminous. Most of the studies have focused on developed countries, while a few have concentrated on developing nations. However, in this study, we review those researches directly linked to our purpose. For example, Mirchandani (2013) employed secondary data and

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**Table 1: History of currency in Liberia**

<table>
<thead>
<tr>
<th>Period</th>
<th>USD</th>
<th>West African pound</th>
<th>Liberian dollar coins</th>
<th>Liberian dollar notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1847-1850s</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1850s-1880s</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1880s-1912</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1913-1943*</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1944-2000**</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2001-present</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

*The West African pound was legal tender during this period, replaced in November 1942 by the USD. **Liberian dollar notes were issued at par with the USD by Charles Taylor during the 1989-1997 civil wars, while “Liberty” dollars was issued in areas controlled by ECOMOG in 1991 to invalidate notes looted from the central bank, thereby creating two local currency zones. USD: United States dollar. Source: Taken from IMF Working Paper WP/09/37 authored by Erasmus et al. (2009)
correlation methodology to study the interaction between key economic variables and ERV. The author realized that EXCs were directly and indirectly correlated with most of the variables examined. Also, Arezki et al. (2014) studied the short and long-run associations between real ERV and commodity price volatility in South Africa. The study observed no significant correlation between real ERV and gold prices in the short-run, but it was observed that gold price volatility was influenced by real effective EXC in the long-term. In another research, Shehu et al. (2012) scrutinized the interaction happening among economic growth, trade movements and ERV in Nigeria. Their Granger causality test results disclosed the existence of causal association of ERV with exports, imports and GDP in Nigeria.

Olowe (2009) used monthly data series from January 1970 to December 2007 and employed the GARCH model to examine the performance of the Nigerian foreign exchange environment. The outcomes rejected the null hypothesis of leverage effect from all the employed asymmetry models. Still in Nigeria, Adeoye and Atanda (2011) used ARCH and GARCH models to gauge the behavior of volatility shocks in the nominal and real EXCs. The authors observed the existence and persistence of volatility shocks in the real and nominal EXCs of the Naira and USD monthly data series from 1986 to 2008. In another study, Bala and Asemota (2013) used monthly time series data of EXC return from the period 1985-2011 for the Naira/USD return and from 2004 to 2011 for the Naira/British Pounds and Naira/Euro returns. By applying the variants of GARCH models, the authors found the USD as the most volatile variable, but they observed the British Pound to be the least volatile. Except for their GARCH models that had volatility breaks, all their asymmetric models rejected the existence of a leverage effect.

Apart from the aforementioned studies, there have been other researches that have evaluated the effect of ERV on trade performance. For instance, Franke (1991), and Sercu and Vanhulle (1992) maintain that volatility in EXC can positively influence trade volume. This result is supportive of the argument that highly volatile EXC regime offers greater opportunity for profits (income effect) and, therefore, ERV, to some degree can boost trade. Hence, this implies that an extremely volatile EXC regime has the propensity to improve trade performance because investors will prefer to export more with the hope of making profits, thereby increasing the volume of trade. Contrary to this thesis, some researchers (Hooper and Kohlhagen, 1978; Cote, 1994; Arize et al., 2000) argue that ERV weakens trade performance. Hence, in line with the theory of risk-aversion, this implies that trade may be negatively associated with ERV.

Another string of the literature has popularized the interactions between EXC and economic growth and has arrived at conflicting results. For example, Edwards and Levy-Yeyati (2003) argue that countries with more flexible EXC regime experience higher level of economic growth. These authors are at the front of the argument that acceleration of economic growth is considerably linked with real EXC depreciation. Also, Akpan (2008) examined the foreign exchange market environment in Nigeria from 1970 to 2003 and observed that EXC and economic growth were positively correlated. Similarly, Asher (2012) studied the effect of EXC variation on economic output in Nigeria for the period 1980-2010. The outcomes revealed that real EXC was also positively associated with economic growth. In the same line of study, Azeez et al. (2012) and Obansa et al. (2013) also investigated the connection between EXC movement and economic output in Nigeria. In general, these authors have found strong and positive association between EXC movement and the level of economic growth. They claim that EXC liberalization promotes economic growth in Nigeria.

Despite the argument that EXC behaves positively towards economic activity, there are other studies which demonstrate that fluctuation in EXC has destructive impact on economic growth. One research at the forefront of the debate it the one by Bosworth et al. (1995), which argues that real EXC fluctuation impedes economic growth and shrinks economic productivity. Also, Aghion et al. (2009) support this thesis and debate that real EXC

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**Figure 1:** United States dollars-Liberian dollars exchange rate regime in Liberia, 1980M1-2017M1

fluctuation can shrink economic output in countries with higher levels of financial development.

In view of the literatures presented above, this study therefore anticipates to lend its voice on the ERV and economic growth nexus within the framework of Liberia. Although the literatures have concentrated on the thesis of EXC behavior on macroeconomic activities, and on currency devaluation and appreciation, as well as trade performance, the macroeconomic performance in Liberia has distinguishing features and warrants scrutiny. Hence, the uniqueness of this study lies in the use of time series data to examine the leverage effects of EXC variation on economic development in Liberia, amid pressure on the Liberian government to stabilize the EXC regime, to lower import taxes, and to control prices.

### 3. DATA AND STATISTICAL PROPERTIES

#### 3.1. The Examined Data and Sources

Although economic growth, consumer participation, and the general financial situations vary globally, the overall macroeconomic variables within individual countries remain the same. Consequently, this study examines the connections between ERV and key macroeconomic variables in Liberia from 1980 to 2015, using econometric modeling techniques. 1980 has been chosen as the start off year due to availability of usable data for analysis. Although Lin and Wesseh (2013a; 2013b) mention that the use of higher frequency data leads to better approximation and analysis, the study period and data frequency implemented in this paper are based on availability grounds. Put another way, although daily or weekly data frequency is more appropriate for capturing the effect of ERV on the Liberian macro-economy, the real gross domestic product (RGDP), consumer price index (CPI), and TB data for Liberia are only available at yearly frequency, thus constraining the writers to use yearly data. The selected sample period and data enable the authors to perform sensitivity study of the Liberian economy, taking into account the past civil crisis that lasted from 1989 to 2003, the Ebola crisis of 2014-2015, and the current drawdown in the Liberian economy, amid plans by government to raise economic growth by double digits.

The examined macroeconomic indices include RGDP, CPI, official EXC and TB. These macroeconomic variables have been chosen because they represent the main channels through which EXC fluctuations can directly impact economic activities (Ocran, 2010; Brun-Aguerre et al., 2012; Fatai and Akinbobola, 2015). For example, EXC variable is very important because the sales and purchases of commodities in Liberia are done in both the USD and LRD. Therefore, fluctuations in EXCs can impact commodity prices, which can in turn affect sales of goods and services, and the export and import of goods and services as well as transportation, communication and agricultural activities, among others.

The datasets have been transformed to avoid spurious and misleading results. The real GDP data is the inflation-adjusted measure of GDP at constant prices (2010 = 100). Liberia’s real GDP data are from the world development indicators database, and are expressed in million USD. The official EXC data is the annual average of the official rate of exchange between the LRD and a unit of the USD. Official EXCs data are from HistData.com and are expressed as ratio of LRD to one unit of the USD (LCU/USD). The CPI data is the annual average of consumer’s prices, not end-of-period data. The CPI measures changes in the prices of goods and services that households consume. Such changes affect the real purchasing power of consumers, especially their incomes and welfare. In this paper, CPI is assigned a value up to 100. Any value above this threshold implies extreme inflationary situation. The CPI data is from the database of the Central Bank of Liberia. The TB data was computed by taking the natural logarithmic difference between total goods exports and the total goods imports. That is:

$$TB_t = \ln(TEXP_t) - \ln(TIMP_t)$$ for $t \in \{1, \ldots, T \text{ years}\}$

The export and import data were taken from the international monetary fund database and are expressed in million USD.

#### 3.2. ERV Estimation

A number of studies have documented various methods of computing ERV. For example, Anderton and Skudelnny (2001) compute ERV by taking the quarterly variance of the weekly nominal EXC, whereas Zubair and Jega (2008) evaluate ERV by taking the standard deviation of each series through their sample. Gujarati (2003) propose that the use of mean-adjusted and the variance of each series in a sample can adequately represent ERV. In a different study involving oil price volatility estimation, Hamilton (2003) argue that incorrect functional form specification of oil price shocks could arise and contribute to inconsistent empirical results and misleading interference from oil price-macro-economy study. As a result he suggests that oil price volatility should be calculated as the logarithmic difference of the annual average prices. Since EXC fluctuates much as oil price does, in line with Hamilton’s (2003) method, this study computes the annual changes (volatility) in official EXCs by taking the natural logarithmic difference between two EXC values. Hence,

$$ERV_t = \ln(EXC_{t}) - \ln(EXC_{t-1})$$ for $t \in \{1, \ldots, T \text{ years}\}$

Where, $ERV_t$ is exchange rate volatility; $\ln(EXC_t)$ is the natural logarithm of official exchange rate at time $t$; and $\ln(EXC_{t-1})$ is the natural logarithm of official exchange rate at time $t-1$. An interpretation of the correlation between $EXV_t$ and RGDP is that there exists a linear relationship between exchange rate volatility and real GDP. The variables are described in Table 2.

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2 Citizens of Liberia gathered in the major cities to protest against the high tax levied by the central government on export and commercial activities and due to the persistent increase in exchange rate as well as increases in the general level of commodity prices (http://www.frontpageafricaonline.com/index.php/business/3232-mask-protest-looks-over-tattered-liberian-economy/).

3 The government of Liberia has crafted the agenda for transformation, aiming at raising economic growth by double digits (http://allafrica.com/download/resource/main/main/idates/000080846:1b0f1d46c3e2c30ce1568f76d4c67b9.pdf).

4 The drop in Liberia’s key export commodity prices and attending effects of the Ebola outbreak reduced Liberia’s economic growth in 2015 by 0.4% (https://www.afdb.org/en/countries/west-africa/liberia/liberia-economic-outlook/).
In order to have a broader sense of the study variables, plots of all examined variables are provided in Figure 2. From Figure 2, it is clear that ERV has been the most volatile variable in Liberia, thereby exerting pressure on trade. The effect is seen in the negative TB for most part. This effect has also been transmitted into the general price level, as reflected by increasing CPI values.

Additionally, when the ERV data are separated into volatility increase and volatility decrease, the plots in Figure 3 can be obtained. Figure 3 illustrates that the EXC regime in Liberia has not been smooth, rather it has been rising and falling.

3.3. Descriptive Properties
An initial analysis of the data demonstrates that it consists of 36 observations. The mean, standard deviation, skewness, ketosis, and Jacque-Bara statistics of the data are reported in Table 3. The outcomes show that the means of all variable are not zero. Additionally, the sample standard deviations fall in the range of 0.612 and 395.600, indicating that real GDP is the most volatile variable while ERV is the least volatile. The outcomes indicate that CPI is skewed to the right, while real GDP, TB, and ERV are skewed to the left. Moreover, ERV has the highest level of kurtosis, meaning that extreme changes occur more often for ERV. Besides, the Jarque-Bera statistic rejects normality at the 10% significance level for all variables, except real GDP, suggesting that the distributions of their series have thicker tails than normal.

4. METHODOLOGY
4.1. Modeling Framework
The non-stationarity of most time series macroeconomic variables may result in spurious results, unreliable inference and even misleading recommendations and conclusions. Therefore, the need to apply techniques in econometric analysis to avoid these modeling biases has become imperative. To begin with, we start by conducting unit-root test to examine the stationarity property of our variables in order to avoid non-spurious results. If a linear combination of the variables is integrated of the same order, then we conduct cointegration test to capture long-run dynamic and equilibrium relationships between co-integrating variables. If they are not cointegrated of the same order, the standard practice is to directly run the Granger (1969) causality test to gauge cause and effect relationship among the variables. This is implemented in an U-VAR model. However, if we find that our variables are

![Figure 2: Plots of the examined variables](image)

Source: Author’s construction

Table 2: Variables and units of measure

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol used in this paper</th>
<th>Unit of measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real gross domestic product</td>
<td>RGDP</td>
<td>US dollars</td>
</tr>
<tr>
<td>Exchange rate volatility</td>
<td>ERV</td>
<td>US dollars</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>CPI</td>
<td>Index</td>
</tr>
<tr>
<td>Trade balance</td>
<td>TB</td>
<td>US dollars</td>
</tr>
</tbody>
</table>

Table 3: Statistical properties of the examined variables

<table>
<thead>
<tr>
<th>Variables*</th>
<th>Statistic</th>
<th>RGDP</th>
<th>CPI</th>
<th>TB</th>
<th>ERV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>760.377</td>
<td>114.846</td>
<td>−0.415</td>
<td>−0.128</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>395.600</td>
<td>114.613</td>
<td>0.684</td>
<td>0.612</td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.254</td>
<td>0.896</td>
<td>−1.015</td>
<td>−5.490</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.629</td>
<td>2.503</td>
<td>3.732</td>
<td>31.794</td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>3.118</td>
<td>5.047712***</td>
<td>6.793941**</td>
<td>1384.958*</td>
<td></td>
</tr>
</tbody>
</table>

*RGDP: Real gross domestic product, CPI: Consumer price index, TB: Trade balance, ERV: Exchange rate volatility. ***Indicate significance at the 1%, 5%, and 10% levels, respectively. Should be around 3 for a normal series.
cointegrated, we then apply a vector error correction model (VECM) to measure short-run as well as long-run dynamic behavior among the variables (Gujarati, 2004).

In general, the modeling framework is summarized in Figure 4.

Based on the econometric modeling procedures outlined in Figure 4, this study employs an unrestricted VAR technique based on the following multivariate model to estimate the effect of ERV and key macroeconomic variables on Liberia’s economic output.

\[ RGDP_t = f(CPI_t, TB_t, ERV_t, ERI_t, ERD_t) \]  

(3)

Where, \( RGDP_t \) is the dependent variable, \( f \) is the functional form specification, \( CPI_t, TB_t, ERV_t, ERI_t, \) and \( ERD_t \) are regressors which have been previously defined.

4.2. Unit Root Test

The process of testing for unit root in time series data is very important. A unit root test is conducted to check if a time series variable is nonstationary by applying an autoregressive model. Stationarity property helps to determine the order of integration and often aids in selecting an econometric model. In recent years, several statistical and econometric methods have evolved, which test the stationarity property of time series data. In this study, we begin by conduction the Augmented Dickey–Fuller (Dickey and Fuller, 1979) unit-root test. The testing procedure for the ADF test follows the model:

\[ \Delta y_t = \alpha + \beta_1 y_{t-1} + \gamma_1 \Delta y_{t-1} + \ldots + \gamma_k \Delta y_{t-k-1} + \epsilon_t \]  

(4)

Where, \( \alpha \) is an \((n \times 1)\) intercept vector, \( \beta \) is an \((n \times n)\) time trend coefficient matrix; \( k \) is the lag order of the autoregressive process; \( \gamma \), \( \delta \), and \( \delta_1 \), are coefficients to be estimated; and \( \epsilon \) is the \((n \times 1)\) generalization of a white noise process. If \( \alpha = \beta = 0 \), then we have a random process, but imposing the constraint \( \beta = 0 \) leads to a random process with a draft. By including lags of the order \( k \) the ADF process allows for higher-order autoregressive processes, which indicates that the lag length \( k \) has to be determined when implementing the test. In this study, the optimal lag length for the ADF unit-root tests is based on the Schwarz (1978) information criterion (SIC). The unit-root test is conducted under the null hypothesis that \( y = 0 \) against the alternative of \( y < 0 \). Once the test statistic is calculated, its value is compared to the appropriate critical value of the Dickey–Fuller test. If the \( t \)-statistic is less than the critical value (more negative), then we reject the null hypothesis and conclude that the series has no unit-root; meaning that it is stationary.

However, it is now understood that the outcomes of ADF test are lag dependent. According to Agiakoglu and Newbold (1992), the ADF test often tends to under-reject the null hypothesis of no unit-root. Therefore, in confirmation to the outcomes of the ADF test, we also apply the Phillips–Perron (PP, 1988) unit-root test in this study. The PP test is implemented by means of a nonparametric method, which controls for serial correlation in the unit-root process. The optimal bandwidth selection of the PP-test is based on the Newey and West (1987) method and the test probabilities are computed by asymptotic Chi-square distribution. Like the ADF method, the PP unit-root test tests the null hypothesis that a time series \( y \) is I(1).

Finally, we also implement the Dickey Fuller generalized least square test to check for unit root in our sample. The test was developed by Elliott, Rothenberg and Stock (ERS) (Elliott et al., 1996) by modifying the ADF. The Dickey Fuller generalized least square method dominates other existing econometric unit root tests in terms of power. It locally de-trends or de-means data series to estimate the deterministic parameters. Then, it uses the transformed data to conduct a standard ADF unit root test. This technique assists in removing the means and linear trends of series that are near the non-stationary region.

Like the ADF and PP tests, the DF-GLS (ERS) unit root test tests the null hypothesis of unit root against the alternative of no unit root. In this paper, the lag length selection of the DF-GLS (ERS) test is based on the SIC and the test diagnostics is based on the \( t \)-statistic and the Mackinnon (1996) critical values. If the absolute value of the \( t \)-statistic is less than the Mackinnon (1996) critical value, we cannot reject the null hypothesis. Alternatively, if the absolute value of the \( t \)-statistic is greater than the absolute value of the Mackinnon (1996) critical value, we can reject the null hypothesis and conclude that our series is stationary. Results for ADF, PP and DF-GLS (ERS) unit-root tests are reported in Table 4.

4.3. Cointegration Test

Cointegration test helps to establish the choice of the appropriate econometric model. There are three main methods used to test for cointegrating relationships among variables in a model: Engle and Granger (1987) two-step method, Johansen cointegration test, and the Phillips–Ouliaris (1990) cointegration test. The Engle and Granger (1987) two-step estimation technique is a single-equation technique for assessing cointegration relationships. It is, however, based on a restraining assumption of a single cointegration correlation that can be estimated by ordinary least squares method. This method tends to fail when there are more than two variables, because there may also be more than one equilibrium association in the model, which causes problems in estimating the cointegration rank of the variables.

5 The Akaike (1973) information criterion (AIC), Schwarz (1978) Bayesian criterion (SBC) (or Schwarz information criterion, SIC), and the Hannan and Quinn (1978) criterion (HQC) are used to determine optimal lag order for the autoregressive process.

6 This test is non-symmetrical, so we do not consider an absolute value.

7 The default setting of the test is Bartlett (1937) kernel.
Unlike the Engle and Granger test, the maximum likelihood approximation technique developed by Johansen (1988; 1995) is often applied in estimating long-run equilibrium associations among time series variables. Contrary to the Engle-Granger’s single-equation model, the Johansen’s maximum likelihood estimation method efficiently includes the short-run dynamics in the structure of the long-run equilibrium model. The main advantage of using the Johansen’s VAR estimation technique lies in the testing and estimation of the multiple long-run equilibrium relationships. Besides, the testing procedures allow one to test economic hypotheses by means of linear restrictions in cointegration space (see, for example: Johansen and Juselius 1990; 1994). However, the main weakness in applying the Johansen’s modeling technique lies in its asymptotic (i.e. large sample) properties.  

\[ \text{Higher requirements for large sample size in Johansen’s cointegration method procedure usually necessitates the use of quarterly or monthly} \]
The Phillips–Ouliaris test demonstrates that the residual-based unit root tests applied to the estimated cointegrating residuals do not have the normal Dickey–Fuller distributions under the null hypothesis of no-cointegration. Due to its spurious regression nature under the null hypothesis and since our sample size is not very small we implement the Johansen cointegration method to examine the long-run equilibrium relationships among our variables.

4.4. Asymptotic Linear Tests for Granger Causality

Real GDP, CPI and TB are shown to maintain unit root processes at level, so they are presented in their first-differenced form. This is necessary because conventional Granger causality test based on unrestricted VAR follows the assumption of stationarity of the variables in the VAR model. Since the data do not show cointegration relationship between real GDP, CPI, TB and ERV, we apply the following unrestricted VAR model to test for linear Granger causality from ERV to economic growth, CPI, and TB:

\[ Y_t = b_0 + \sum_{i=1}^{k} \gamma Y_{t-i} + \sum_{i=1}^{k} \delta_i X_{t-i} + D + \epsilon_t \]  

(5)

In Equation (6), \( \beta \) and \( \delta \) are the \( p \times (r + \pi) \) matrices of the autoregressive coefficients for \( i, 1, 2, \ldots, k \) and \( \xi \) is the \( (r + \pi) \times 1 \) innovation of a white noise process. \( D \) denotes dummies that are incorporated to capture structural changes. Granger (1969) causality is said to exist when the values of \( Y_t \) have explanatory power in a regression of \( Y_t \) on lagged values of \( Y_t \) and \( X_t \). However, if lagged values of \( X_t \) do not have such explanatory power for any of the other variables in the system, then \( Y_t \) is considered weakly exogenous to the system. For each equation in the VAR specification in Equation (5), Wald \( \chi^2 \) statistics are used to test the joint significance of all lagged endogenous variables. Moreover, Wald \( \chi^2 \) statistics provide information about whether an endogenous variable can be treated as exogenous or not. The optimal value of \( k \) based on AIC, SIC and HQC pointed to 2.

In order to examine the response to ERV, we transform the VAR system in Equation (5) to its moving average representation and then use it to measure the impulse response and forecast error variance decomposition. The moving average specification is expressed as:

\[ Y_t = \rho + \sum_{i=0}^{m} \varphi_i Y_{t-i} \]  

(6)

Where, \( \varphi \) is the identity matrix and \( \pi \) is the average of the process.

5. RESULTS AND DISCUSSIONS

5.1. Results

5.1.1. Stationarity analysis

ADF, PP and DF-GLS stationarity tests results are detailed in Table 4. Panel A contains results of the tests involving intercept, trend and intercept, and neither intercept nor trend for the ADF, PP and DF-GLS unit root tests for all series at level. Panels B contain results for the first differenced series. Analysis of the outcomes shows that, for all three cases (intercept, trend and intercept, and neither intercept nor trend), real GDP and CPI maintain unit roots at level. But, for the TB variables the decision of unit process is inclusive at level. However, by performing additional tests, as reported in Panel B, all the nonstationary series become stationary after taking first difference or I(1). Hence, since all our variables become I(1) after taking first difference, it makes cointegration estimation based on Johansen’s maximum likelihood method achievable.

5.1.2. Johansen’s cointegration analysis

Because our variables are I(1) variables, it becomes convenient to test for cointegration relationships by means of Johansen’s cointegration method. The results of Johansen cointegration test are reported in Table 5. The lag length structure based on the LR, FPE, AIC, HQ and SICS points to 2. The Johansen’s method offers two unrestricted cointegration rank tests, namely the trace and maximum eigenvalue tests. In Panel A, the results for the trace test are shown. The outcomes show that our four variables are not cointegrated at the 5% significance level, because the trace statistic of 43.13 is less than the critical value 47.84. Moreover, the P value of 12.93% rejects the null hypothesis of no cointegration, indicating that our variables are not cointegrated.

Similarly, the results for the maximum eigenvalue test are reported in Panel B. In conformity with the trace test result in Panel A, the maximum eigenvalue test result demonstrates that the variables are not cointegrated. This decision is based on a lower maximum eigenvalue test result of 23.91 as compared to a critical value of 27.58. Additionally, the null of no cointegration is rejected at the 5% significant level due to the probability of 13.78%, thus providing evidence against the presence of a cointegration relationship between ERV and Liberia’s macroeconomic variables. Since cointegration denotes long-run Granger causality, the absence of cointegration does not eliminate the presence of short-run causal associations. This implies that ERV does not affect the level of economic activities in the long-run in Liberia. To fortify the policy relevance of the paper, short-run estimates of the elasticity of real GDP with respect to ERV and other macroeconomic indices are provided. Moreover, since both tests results confirm that the variables are not cointegrated, we estimate an U-VAR model to check for cause and effect relationships amount the variables. Before running the U-VAR, additional tests for residual autocorrelation and heteroscedasticity are performed.

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9 The mathematical details for the Johansen’s test are presented in the Appendix A.

10 One must not forget that the measures for exchange rate volatility are stationary transformations of exchange rate, which is an I(1) variable.

11 The lag length is chosen based on an amalgam of techniques—the Akaike (1973) information criterion (AIC), Schwarz (1978) information criterion (SBC), and the Hannan–Quinn information criterion (HQC), final prediction error (FPE), and the sequential modified likelihood ratio (LR) test statistic.
to check whether there is no omitted variable bias in the system. The results of all tests point to the absence of autocorrelation and heteroscedasticity in the system.

5.1.3. Short-run elasticity
Short-run elasticities of economic activities in Liberia are reported in Table 6. The outcomes show that CPI and TB are significantly positively correlated with real GDP in the short-run. The result shows that appreciation in ERV causes improvement in real GDP, albeit not significant. Since correlation does not simply suggest causation, clarity is obtained from causality analysis.

5.1.4. Causality analysis
Causality test results based on Granger (1969) linear asymptotic model are reported in Table 7. Similarly to short-run elasticity result in Table 6, the unrestricted VAR model demonstrates that there is no causality running from ERV to Liberia’s real GDP in the long-run. On the one hand, there is unidirectional causality running from real GDP to CPI and ERV and from CPI to TB in Liberia. Additionally, the results show that ERV also Granger causes CPI in Liberia, thus suggesting possibilities of significant linear relationship in the ERV – CPI interaction. On the other hand, there exists no causality from TB to any of the variables. However, findings from short-run elasticity that TB and CPI can positively and significantly influence the level of Liberia’s real GDP in the short-run do not coincide with causality results. This is not surprising because correlation does not simply imply causation.

5.1.5 Impulse response analysis
So far, we have met the criteria for examining the interaction between variables. Nevertheless, the applied models have not said anything definite about how the variables react to each other. Consequently, it is important to determine the impulse response functions (IRF). These are dynamic simulations that trace the response to a one-period shock in the innovation or reaction of an endogenous variable over time to a given shock (impulse). The IRF is applied based on the moving average representation in Equation (6) with dummy variables included to capture structural breaks. Given that the residuals may be instantaneous correlated since the shocks are not likely to occur in isolation, the variance-covariance matrix of the VAR was factorized using the Cholesky decomposition technique proposed by Doan (1992). In order to test the significance and certainty of an impulse response test, it is useful to estimate a confidence interval around the IRF.

In the impulse response graphs, the middle lines indicate the IRFs while the bars represent confidence intervals. Hence, when the horizontal line falls within the confidence interval, the null hypothesis that there is no effect of ERV on other macroeconomic variables cannot be rejected.

Figure 5 documents the orthogonalized IRF for the responses of Liberia’s macroeconomic variables to ERV. The response of real GDP to ERV rises from zero in the 1st period and then returns to zero during the 2nd period. It then increases up to the 6th period and then decreases up to the 10th period. For CPI, the response fell from zero during the 1st period and remained negative throughout the entire periods. The response of TB to ERV was negative up to the 4th period, after which it remained positive throughout.

In Figure 6, one standard deviation of shock to CPI causes an increase in real GDP, but positive and negative changes in other macroeconomic variables. The response of the real GDP to shocks of TB decreases as presented in Figure 7 but shows that the other variables respond positively and negatively.

IRFs based on one standard deviation shock to real GDP are presented in Figure 8. Clearly, CPI responds positively and significantly up to the 6th period and become negative thereafter. The response of TB is positive throughout, whereas ERV responds negatively and positively.

In general, the confidence interval bands suggest that the IRFs were fairly accurate, as they followed the responses nicely throughout.

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Table 5: Results of Johansen’s unrestricted cointegration rank test

<table>
<thead>
<tr>
<th>Hypothesized No. of CE (s)</th>
<th>Eigenvalue</th>
<th>Trace statistic</th>
<th>5% critical value</th>
<th>Probability.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None*</td>
<td>0.526351</td>
<td>43.13768</td>
<td>47.85613</td>
<td>0.1293</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.249686</td>
<td>19.22444</td>
<td>29.79707</td>
<td>0.4769</td>
</tr>
<tr>
<td>At most 2</td>
<td>0.200544</td>
<td>10.03199</td>
<td>15.49471</td>
<td>0.2783</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.085773</td>
<td>2.869635</td>
<td>3.841466</td>
<td>0.0903</td>
</tr>
</tbody>
</table>

Panel B: Unrestricted cointegration rank test (maximum Eigen value)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE (s)</th>
<th>Eigen value</th>
<th>Max-Eigen statistic</th>
<th>0.05 critical value</th>
<th>Probability.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None*</td>
<td>0.526351</td>
<td>23.91324</td>
<td>27.58434</td>
<td>0.127</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.249686</td>
<td>9.192448</td>
<td>21.13162</td>
<td>0.8166</td>
</tr>
<tr>
<td>At most 2</td>
<td>0.200544</td>
<td>7.162352</td>
<td>14.2646</td>
<td>0.4701</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.085773</td>
<td>2.869635</td>
<td>3.841466</td>
<td>0.0903</td>
</tr>
</tbody>
</table>

*Denotes rejection of the hypothesis at the 0.05 level. **MacKinnon et al. (1999) P values. CEs: Cointegrating equation.

Table 6: Short-run elasticities

<table>
<thead>
<tr>
<th>Regressors</th>
<th>RGDP = CPI + TB + ERV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>7.129***</td>
</tr>
<tr>
<td>CPI</td>
<td>3.842***</td>
</tr>
<tr>
<td>TB</td>
<td>0.964***</td>
</tr>
<tr>
<td>ERV</td>
<td>1.072</td>
</tr>
</tbody>
</table>

*Values in table are t-statistics. **Indicates rejection of the null hypothesis at the 1% level of significance. RGDP: Real gross domestic product, CPI: Consumer price index, TB: Trade balance, ERV: Exchange rate volatility.
5.1.6. Variance decomposition analysis

Here, the impact of ERV is analyzed by using the forecast error variance decomposition based on the moving average specification in Equation (6). This is done by examining the proportion of the forecast error variance of a variable that is attributable to its own innovations and other variables in the system. Since the purpose of this study is to demonstrate how the Liberian economy responds to ERV, only variance decompositions for Liberian macroeconomic variables attributable to ERV are reported. The outcomes in Table 8 show that innovations to real GDP can cause a 0.45% fluctuation in ERV in the 1st period and a 9.29% in the 10th period in Liberia. However, changes in CPI account for between 50.05% – 43.11% of the variation in ERV for the time periods under consideration (i.e., 1st, 5th and the 10th period). In general, the result shows that the shocks to real GDP and TB cause increasing fluctuations in ERV in the long-run but a decreasing effect due to shock in CPI in the long-run in Liberia. Clearly, these results are consistent with those reported previously.

5.2. Discussions

The small domestic market and the dual currency situation has been a major impediment to economic growth in Liberia13. As part of efforts to expand the domestic market and alleviate poverty, the government of Liberian and its foreign partners crafted a Poverty Reduction Strategy, aimed at raising the economy from an average 6% growth rate since 2008 to double-digits. Given Liberia’s huge dependence on imported products to meet its domestic needs for food and other necessities, weak control of monetary policy, low development of manufacturing sector and poor state of technological progress, it is surprising that CPI and TB have positive short-run influences on real GDP in Liberia. This is especially reasonable given that imports have exceeded exports in Liberia; thereby giving a negative TB. However, finding that ERV has not impacted economic growth in the short-run is ambiguous; given the fact that the raw EXC data shows extreme fluctuations in EXC regimes in Liberia (Figure 1). Notwithstanding, as Liberia’s current control of monetary policy becomes more stringent and efforts to boost the manufacturing sector and domestic production of goods materialize, chances are that TB and CPI are not likely to negatively influence Liberia’s economic growth. Hence, the findings that ERV, CPI and TB do not affect the level of economic activities in the long-run are reasonable.

On the other hand, finding that ERV affect CPI in the long-run highlights an important factor that characterizes the Liberian economy, i.e. high dependence on remittances from abroad and the use of the USD in the domestic market, which put pressure on the LRD; despite the country being rich in gold, iron ore, diamond, timber, and other resources14. In fact, an astounding World Bank report on global remittances notes: “Liberians in the Diaspora out-beat per GDP other Africans living abroad by remitting $378 million in 2012, an increase of $18 million from 2011 remittance of $360 million, a more than 31% of global remittances and third of

13 Other key factors include, lack of adequate infrastructure, high transportation costs, poor trade links with neighboring countries and the high dollarization of the economy.
Liberia’s GDP in 2012.” This puts the Liberian government under moral and economic obligation to forge a working partnership with its Diaspora community.

Most interestingly, a few important factors must be taken into consideration when interpreting these results for Liberia. First, there has been a period of civil war starting from 1989 to 2003. The war destroyed much of the economy. As a result, many business people fled Liberia leading to serious capital flight. While some of these people have returned, it is obvious that many will not return. Second, local manufacturing, mainly foreign owned, has been very small in scope. Third, Liberia is highly import-dependent. These factors combined have placed Liberia’s current output far below the potential level at which it could be positively affected by TB.

### 6. CONCLUSION AND POLICY IMPLICATIONS

This study analyzes the effect of ERV on key macroeconomic variables in Liberia. The results from ADF, PP, DF-GLS (ERS) unit-root tests indicate the stationarity of Liberia’s macroeconomic variables at first difference. Cointegration analysis by Johansen’s maximum likelihood method shows the absence of cointegration. Consequently, the study applies an unrestricted VAR model to gauge the interaction among the variables.

The empirical findings show no significant relationship between ERV and Liberia’s real GDP in the short and long-run. Notwithstanding, results from the unrestricted VAR model show that ERV Granger causes CPI and that CPI Granger causes TB in Liberia. Results from short-run elasticity show that CPI and TB have both influenced economic growth in the short-run in Liberia. Findings from variance decomposition show that innovations to real GDP and TB cause increasing fluctuations in ERV in the long-run but a decreasing effect due to shock in CPI in the long-run in Liberia.

Although there has been no significant relationship between ERV and real economic output in Liberia, this study argues that it is necessary for Liberian policymakers to exert stronger monetary policy control and higher level of technological innovation to ensure the existence of single currency regime in the country. Moreover, the government needs to regulate the use and exchange of the USD by limiting it only to banking institutions and support the use of only the LRD on the domestic market. This also implies that foreign remittances should be received in LRD. This will help to ease the pressure on the LRD and stabilize the economy.

Finally, a more general insight from the study is that there is a need to increase technological progress and boost domestic production in order to increase export and reduce imports. This will help to offset the negative TB and improve Liberia’s real GDP. Moreover, since enlargement of the economy’s productive base is also key to limiting any potential effects of trade, Liberian policymakers must also forge a working partnership with its Diaspora community in order to ensure that economic impact of remittances is translated into the improvement of the living condition of Liberians. As development would rely heavily on the magnitude of trade happening, sufficient investment incentives for remittance recipients will have to be created. Given that this would involve higher capital expenditures for potential investors from the start, price-based policies alone might not be fully effective.

### Table 7: Causality based on Granger (1969) linear asymptotic model

<table>
<thead>
<tr>
<th>X (regressor): Direction of causality (→)</th>
<th>Y</th>
<th>RGDP</th>
<th>CPI</th>
<th>TB</th>
<th>ERV</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGDP</td>
<td>-</td>
<td>0.005***</td>
<td>0.216</td>
<td>0.061***</td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>0.957</td>
<td>-</td>
<td>0.033***</td>
<td>0.614</td>
<td></td>
</tr>
<tr>
<td>TB</td>
<td>0.719</td>
<td>0.584</td>
<td>-</td>
<td>0.288</td>
<td></td>
</tr>
<tr>
<td>ERV</td>
<td>0.980</td>
<td>0.000***</td>
<td>0.494</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

*All figures in the table are P values. ***Indicates rejection of the null hypothesis at the 10% significance level. RGDP: Real gross domestic product, CPI: Consumer price index, TB: Trade balance, ERV: Exchange rate volatility

### Table 8: Variance decomposition analysis

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Period</th>
<th>Exchange rate volatility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGDP</td>
<td>1</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.82</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>9.29</td>
</tr>
<tr>
<td>CPI</td>
<td>1</td>
<td>50.05</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>44.29</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>43.11</td>
</tr>
<tr>
<td>TB</td>
<td>1</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.69</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>6.70</td>
</tr>
</tbody>
</table>

*Factorization done by Cholesky decomposition method. RGDP: Real gross domestic product, CPI: Consumer price index, TB: Trade balance, ERV: Exchange rate volatility

### Figure 8: Impulse response functions of shocks to real gross domestic product
especially when the investment is outside the realm of financial assets. It would therefore be necessary to implement strategies intended to minimize capital expenditures related to setting up a new business. For example, capital subsidies and lower taxes on capital expenditures in combination with price-based policies should be implemented. Lastly, it will be important for the government to maintain tight controls on the prices of goods and services. This will help to correct the imbalances in commodity prices and ease the economic hardship on the common citizens.

**REFERENCES**


Appendix A: Mathematical details of the Johansen and Juselius (1990) and Johansen (1995) cointegration test

Let a VAR of order; be defined as follows:

\[ y_t = \beta y_{t-1} + \gamma x_t + \delta x_t + \varepsilon_t \]  \hspace{1cm} (A-1)

Where, \( y_t \) is an \( (n \times 1) \)-vector of nonstationary I(1) variables; \( x_t \) is \( (k \times 1) \)-of deterministic variables; \( \beta \) through \( \delta \) and \( \gamma \) are coefficients to be estimated; and \( \varepsilon_t \) is the \( (m \times 1) \) generalization of a white noise process. Equation (A-1) can be restated as follows:

\[ \Delta y_t = \Pi y_{t-1} + \sum_{j=1}^{q} \Delta y_{t-j} + d x_t + x_t \]  \hspace{1cm} (A-2)

Where, the following can be defined:

\[ \Pi = \sum_{i=1}^{r} \beta_i - I \] and \( \Gamma_j = - \sum_{i=1}^{j} \beta_i \)  \hspace{1cm} (A-3)

The Granger’s representation theorem states that if the coefficient matrix \( \Pi \) has reduced rank \( r < q \), then there exists \( q \times r \) matrices \( \theta \) and \( \gamma \) each having rank \( r \) for which \( \Pi = \theta \gamma' \) and \( \gamma' y_t \) is I(0). The cointegrating rank is \( r \) and each column of \( \gamma \) is the cointegrating vector. The elements of \( \theta \) are the adjustment parameters in the VECM. The Johansen’s technique is to determine the \( \Pi \) matrix from an unrestricted VAR and then test if the restrictions implied by the reduced rank of \( \Pi \) can be rejected.

The five deterministic trend cases applied in the Johansen’s (1995. p. 80-84) cointegration model can be summarized as follows:

1. The is no deterministic trends and the cointegrating equations (CEs) do not have intercepts at level:

\[ H_r(r): \Pi y_{t-1} + \delta x_t = \theta \gamma \gamma' y_{t-1} \]  \hspace{1cm} (A-4)

2. There is no deterministic trends and the CEs have intercepts at level:

\[ H_r(r): \Pi y_{t-1} + d x_t = q (q' y_{t-1} + r_0) \]  \hspace{1cm} (A-5)

3. Linear trends exist in the level data but the CEs only have intercepts:

\[ H_r(r): \Pi y_{t-1} + \delta x_t = \theta \gamma \gamma' y_{t-1} + \rho_1 \] \hspace{1cm} (A-6)

4. Both the level data and the CEs have linear trends:

\[ H_r(r): \Pi y_{t-1} + \delta x_t = \theta \gamma \gamma' y_{t-1} + \rho_1 + \theta \Delta \pi_0 \] \hspace{1cm} (A-7)

5. There exists quadratic trends in the level data and CEs have linear trends:

\[ H_r(r): \Pi y_{t-1} + \delta x_t = \theta \gamma \gamma' y_{t-1} + \rho_1 + \theta \Delta \pi_0 + \pi_1 t \] \hspace{1cm} (A-8)