Power System Stabilizer Based on Global Fuzzy Sliding Mode Control

E. Nechadi and M.N. Harmas

Abstract—Power systems stability is enhanced through a novel stabiliser developed around a fuzzy sliding mode approach. First, sliding mode control is applied to selected operating point based models of a power system separately then fuzzy logic is used to form a global model encompassing the separate subsystems, thus leading to a fuzzy sliding mode power system control. Stability is insured through Lyapunov synthesis. Severe operating conditions are used in a simulation study to test the validity of the proposed method, indicating better performance and satisfactory transient dynamic behaviour.

Index Terms—Power system stabiliser; sliding mode control; sliding surface; fuzzy sliding mode; Lyapunov stability.

I. INTRODUCTION

The need for more reliable power margins and less amount of electro-mechanical oscillations that limit power flow in complex power systems, has imposed the addition of stabilizers coined power system stabilizer PSS as early as the mid 40s, nowadays tagged conventional, classical stabilisers or CPSS[1-3]. Highly non linear, time varying, power systems have been and remain a major challenge to control and power engineers alike.

Effectively, power systems are complex nonlinear systems that often exhibit low frequency oscillations due to insufficient damping caused by adverse operating conditions which can lead the underlying machine to lose synchronism. Power system stabilizers are used to suppress these oscillations and improve the overall stability. Conventional stabilizers, consisting of cascade connected lead-lag compensators derived from a linear model representing the power system at a certain operating point, have long been used to damp oscillations regardless of the varying loading conditions or disturbances. However, this linear model based control strategies often fail to provide satisfactory results over a wide range of operating conditions.

Many alternatives have been put forth since pioneering CPSS not only in stabilizer design but in power model elaboration as well. Further research work led to more appropriate adaptive approaches as in [4-6]. Robust control techniques have been also suggested in effort to circumvent parameters uncertainties effect as well as exogenous disturbances leading to sliding mode based PSS [7-10] and Hinf PSS [11].

Remarkable research effort has been done in the last decade putting forward intelligent fuzzy logic based PSS as well as optimality in adapting to changing operating conditions as in [12-17]. Amid the many interesting schemes suggested, the combination of sliding mode technique and the fuzzy approach capitalising on the free model aspect of the latter and the robustness of the first seem to be promising. Based on work developed by Yu [18] we have elaborated a power system stabilizer using fuzzy logic to amalgamate several sliding mode controlled based linear power system models, obtained for selected operating points.

This paper introduces briefly in the next section the sliding mode control approach used, followed by the third section in which Takagi-Sugeno fuzzy technique is tackled. In section IV the design of the global fuzzy sliding mode stabilizer is undertaken and stability issue addressed. The power system model is presented in the ensuing section followed by simulation and a presentation of results for different operating conditions.

II. SLIDING MODE CONTROL

Sliding mode control is a part of the theory of Variable Structure Control (VSC) which is a control technique relying on a practical high-speed switching between different configurations basically inspired by relay control theory. This variable structure control provides an efficient method for nonlinear plants control.

Reluctant at first, control engineers have shown increased interest in variable structure control as advances in electronic circuitry and computer technology took place making feasible many practical implementations such as in robotics [19-20], or in power system control as in [11,21].

Essentially, VSC utilizes a high-speed switching control law to drive the nonlinear plant’s state trajectory onto a selected designer chosen sliding surface in the state space and to maintain plant’s state trajectories on this surface for all subsequent time. The plant dynamics restricted to this surface represent the controlled system’s behaviour.

Consider a SISO dynamical system described by:

$$
\dot{x}(t) = Ax(t) + Bu(t),
$$

where $x(t)$ is the state vector, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$ and $u(t)$ is the sliding mode control input.

Sliding mode control design comprises two phases: first a switching surface imposing desired dynamics in the sliding mode is elaborated, followed by the design of the
discontinuous control law that drives the system state trajectories towards the switching surface. The switching surface is generally defined as:

\[ s(x) = Cx(t) \]  

(2)

\( C \) is the sliding vector, which can be determined using different available techniques, we will use pole placement in the sliding phase according to Ackerman’s approach [22].

Control law enabling satisfaction of the attraction phase condition (3) and the equivalent control to maintain state trajectories on the sliding surface is typically given by (4) assuming \((CB)\) is non-singular.

\[
\begin{align*}
 s(x) &< 0 \\
 u & = -(CB)^{-1} [CAx + k \text{sign}(s)] ; k > 0
\end{align*}
\]

(3)

\( \| \| \) indicates the euclidean norm, used here to reduce chattering when approaching the equilibrium point.

Background details for sliding mode theory and sliding surface design can be found in [22-24].

III. DESIGN OF FUZZY CONTROL

In our control design procedure, a Takagi-Sugeno fuzzy model is used to represent a global model of the underlying nonlinear plant. The fuzzy model described by fuzzy IF-THEN rules represents local linear input-output relations of the considered nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model.

The global fuzzy model of the system is achieved by fuzzy ‘blending’ the operating point based linear system models [18, 19]. The \( i \)-th rule of the T-S fuzzy models is of the following form:

Rule i:

IF \( z_i(t) \) is \( F_{ii} \) AND … \( z_p(t) \) is \( F_{ip} \), THEN

\[ x(t) = A_i x(t) + B_i u(t), \quad y(t) = D_i x(t) + E_i u(t), \quad i = 1, 2, 3, \ldots, m \]  

(5)

where, \( F_{ip} \) is the fuzzy set and \( m \) is the number of model rules, \( x(t) \), \( u(t) \), \( y(t) \) are respectively the state, the input and the output vectors, \( A_i \in \mathbb{R}^{n_{Ax}}, \quad B_i \in \mathbb{R}^{n_{Bu}}, \quad D_i \in \mathbb{R}^{n_{Dx}}, \quad E_i \in \mathbb{R}^{n_{Ex}} \) and \( z_i(t), \ldots, \ z_p(t) \) are known premise variables that may be functions of the state variables, external disturbances and time. We will use \( z(t) \) to denote the vector containing all the individual elements \( z_1(t), \ldots, z_p(t) \). Each linear consequent equation represent a called a subsystem.

Let \( \mu_i(z(t)) \) denote the normalized fuzzy membership function of the inferred fuzzy set \( F_i \) where

\[ F_i = \sum_{j=1}^{p} F_{ij} \quad \text{and} \quad \sum_{i=1}^{m} \mu_i = 1. \]  

(6)

The global fuzzy state space model is given by:

\[ \begin{align*}
 x(t) &= Ax(t) + Bu(t), \\
 y &= Dx(t) + Eu(t)
\end{align*} \]  

(7)

Where

\[
\begin{align*}
 A &= \sum_{i=1}^{m} \mu_i A_i, \quad B = \sum_{i=1}^{m} \mu_i B_i, \\
 D &= \sum_{i=1}^{m} \mu_i D_i, \quad E = \sum_{i=1}^{m} \mu_i E_i
\end{align*}
\]

(8)

Let us make the assumption that \((A, B)\) of the global system is completely controllable based on the assumed controllability if each subsystem.

IV. DESIGN OF FUZZY SLIDING MODE CONTROL

In their paper X. Yu et al. [18] used a constant sliding surface to developed their remarkable results that we revisit here with our contribution being the use of different sliding surfaces corresponding to different operating points that our application requires, a power system, in order to uphold the desired poles placement. The main drawback in such a system resides in the necessity for a new sliding manifold for each new configuration in order to enable the same pole placement and thus identical dynamic performances.

Theorem 1: Each subsystem of the fuzzy model (5) if we choose the following control \( u_i \) law,

\[
 u_i = -(C_i B_i)^{-1} [C_i A_i x + k_i \text{sign}(s_i)]
\]

then the system is asymptotically stable.

Proof:

Let the Lyapunov function candidate be given as,

\[
 V(x) = \frac{1}{2} s^T(x)s(x)
\]

(10)

Then:

\[
 \dot{V} = s_i^T(x) \dot{s}_i(x)
\]

\[
 = s_i^T(x) C_i(A_i x + B_i u_i)
\]

\[
 = -k_i s_i^T(x) \text{sign}(s_i) \| x \|
\]

and thus: \( \dot{V} < 0 \)

Theorem 2: For the fuzzy system (7), if we choose the following control law for \( i \)-th subsystem \( u_i \):

\[
 u_i = -(C_i B_i)^{-1} [C_i A_i x + k_i \text{sign}(s_i)]
\]

(11)

and if

\[
 C_i B_i = C_j B_j, \quad \text{sign}(s_i) = \text{sign}(s_j) \quad i \neq j
\]

(12)

Then the system is asymptotically stable.

Proof:

Choosing the Lyapunov function candidate to be

\[
 V(x) = \frac{1}{2} s^T(x)s(x)
\]

(13)

and the control law term (14) given here will be designated approach (1)
\[ u = \sum_{i=1}^{m} \mu_i u^i, \quad (1) \text{ approach} \]

Therefore,

\[ \dot{V} = s^T(x) \dot{s}(x) \]
\[ = s^T(x) C(Ax + Bu) \]
\[ = s^T(x) \left( C \sum_{i=1}^{m} \mu_i A_i x + C \sum_{i=1}^{m} \mu_i B_i u^i \right) \]
\[ = s^T(x) \left( \sum_{i=1}^{m} \mu_i (CA_i x + CB_i u^i) \right) \]
\[ = -s^T(x) \sum_{i=1}^{m} \mu_i k_i \text{sign}(s_i) \| s \| \]
\[ = -\sum_{i=1}^{m} \mu_i k_i s_i \text{sign}(s_i) \| s \| \]
\[ \dot{V} < 0. \]

**Theorem 3:** For the global fuzzy system (7), if we choose the following control \( u^k \) for another rule \( k \) (\( i \neq k \))

\[ u^k = - (C_k B_k)^{-1} [C_k A_k x + k_k \text{sign}(s_k) \| s \|] \]

(14)

and if

\[ k_k > k^0_k = \frac{\| CA_k - CB_k (C_k B_k)^{-1} C_k A_k \|}{\lambda_{\min} (CB_k (C_k B_k)^{-1} + (CB_k (C_k B_k)^{-1})^T)}, \]

\[ \lambda_{\min} (CB_k (C_k B_k)^{-1} + (CB_k (C_k B_k)^{-1})^T) \text{sign}(s_k) > 0 \]

(15)

Then the system is asymptotically stable.

**Proof:**

Choosing the Lyapunov function candidate:

\[ V(x) = \frac{1}{2} s^T(x) s(x) \]

Therefore

\[ \dot{V} = s^T(x) \dot{s}(x) \]
\[ = s^T(x) C(Ax + Bu) \]
\[ = s^T(x) \left( C \sum_{i=1}^{m} \mu_i A_i x + C \sum_{i=1}^{m} \mu_i B_i u^k \right) \]
\[ = s^T(x) C \left( \sum_{i=1}^{m} \mu_i A_i x + \sum_{i=1}^{m} \mu_i B_i \left( - (C_k B_k)^{-1} [C_k A_k x + k_k \text{sign}(s_k) \| s \|] \right) \right) \]
\[ = s^T(x) \sum_{i=1}^{m} \mu_i ((CA_i - CB_k (C_k B_k)^{-1} C_k A_k) x) \]
\[ \leq \sum_{i=1}^{m} \mu_i \left\langle \left( CA_i - CB_k (C_k B_k)^{-1} C_k A_k \right) x, x \right\rangle \]
\[ \leq \sum_{i=1}^{m} \mu_i \left( \| CA_i - CB_k (C_k B_k)^{-1} C_k A_k \| \| x \| \right) \]
\[ \leq \lambda_{\min} (CB_k (C_k B_k)^{-1} + (CB_k (C_k B_k)^{-1})^T) \| k_k \| \| s \| \]
\[ \dot{V} < 0. \]

**Theorem 4:** For the global fuzzy system (7), if we choose the global control \( u \) law which will be termed approach (2)

\[ u = - (CB)^{-1} [CAx + k \text{sign}(s) \| s \|] \]

(17)

Where

\[ A = \sum_{i=1}^{m} \mu_i A_i, \quad B = \sum_{i=1}^{m} \mu_i B_i, \]
\[ C = \sum_{i=1}^{m} \mu_i C_i, \quad s(x) = Cx(t) \]

(18)

then the system is asymptotically stable.

Hence, we can write

\[ C = \theta^T \xi(x) \]

(19)

Where \( \theta = [\theta_1, \theta_2, ..., \theta_m] \) is the vector of parameters, \( \xi = [\xi_1, \xi_2, ..., \xi_m]^T \) is the vector of fuzzy basis functions with \( \theta \) being bounded: \( \| \theta \| \leq M_{\theta} \).

If we let:

\[ \dot{\theta} = -\gamma s(x) \xi(x) \]

(20)

Where

\[ \gamma = \frac{M_{\theta}}{\| \xi(x) \|} \]

(21)

Where \( M_{\theta} \) and \( \gamma \) are positive constants.

**Proof:**

Considering the Lyapunov function candidate:

\[ V(x) = \frac{1}{2} s^T(x) s(x) + \frac{1}{2\gamma} \theta^T \theta \]

Therefore

\[ \dot{V} = s^T(x) \dot{s}(x) + \frac{1}{\gamma} \theta^T \dot{\theta} \]
\[ = s^T(x) (C(Ax + Bu) + \dot{\theta}(x) + \frac{1}{\gamma} \theta^T \theta) \]
\[ = s^T(x) \sum_{i=1}^{m} \mu_i C_i \left( \sum_{i=1}^{m} \mu_i A_i x + \sum_{i=1}^{m} \mu_i B_i \left( - (CB)^{-1} [CAx + k \text{sign}(s) \| s \|] \right) \right) \]
\[ + s^T(x) \theta \xi(x) + \frac{1}{\gamma} \theta^T \theta \]
\[ \leq k_s(x) \text{sign}(s) \| \theta \| \| \theta \| \]
\[ = k_s(x) \xi(x) \|

After some straightforward calculations, we obtain the following...
\[ \dot{V} < -k \text{sign}(s) |x| - (M_\theta + |\theta|) s^2(x) \]

and thus: \( \dot{V} < 0 \).

V. POWER SYSTEM MODEL

The power system model considered in this paper is a fourth order linearized model representing a synchronous machine connected to an infinite bus via a double circuit transmission line. The power system schematic diagram including turbine, transformer, automatic voltage regulator and PSS is shown in fig.1 [25].

![Fig.1 Synchronous machine infinite bus.](image)

A fourth order classic state space representation [9, 25] is given in (22). See appendix for nomenclature.

\[
\dot{x} = \begin{bmatrix}
\frac{D}{M} & -K_i & K_i & 0 \\
\frac{M}{M} & -K_i & K_i & 0 \\
0 & 0 & 0 & 0 \\
0 & \frac{K_i}{T_{\theta}} & \frac{1}{T_{\theta}} & \frac{1}{T_{\theta}} \\
0 & \frac{K_i}{T_s} & \frac{1}{T_s} & \frac{1}{T_s}
\end{bmatrix} \begin{bmatrix}
\dot{x} \\
\dot{\omega} \\
\dot{\theta} \\
\dot{e}_q \\
\dot{e}_{\alpha q}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
K_i \\
- \frac{K_i}{T_s}
\end{bmatrix} u
\]

Where the state variable are expressed as: \( x(t) = [\Delta \omega(t) \Delta \delta(t) \Delta e_q(t) \Delta e_{\alpha q}(t)]^T \).

Note that the six constants \( K_1 - K_6 \) are functions of real power \( P \) and reactive power \( Q \) [2, 25].

The parameters of the single machine infinite bus system are as follows:

- \( x_e = 0.4 \text{p.u} \), \( x_q = 1.55 \text{p.u} \), \( x_d = 1.6 \text{p.u} \), \( x_d' = 0.32 \text{p.u} \), \( D = 0 \),
- \( T_{\theta} = 6 \text{s} \), \( H = 5 \text{s} \), \( T_s = 0.05 \), \( K_i = 50 \), \( V = 1 \text{p.u} \),
- \( P_0 = 0.75 \text{p.u} \), \( Q_0 = 0.015 \text{p.u} \).

VI. SIMULATION

The soundness of the approach was tested and performance as well as robustness tests were conducted and compared to a classic CPSS [12] confirming, through computer simulations, good transient behaviour with the proposed approaches despite severe operating conditions illustrated by the following case studies.

![Fig.4 Speed deviation.](image)
Case 2: Operating conditions change abruptly from light to heavy load condition, i.e. $P$ is changed from 0.75 p.u. to 1 p.u. The simulation results in fig.4 show a better transient performance for the second approach.

Case 3: We now consider the case of the sudden occurrence of heavy reactive power causing a change in $Q$ from the light value of 0.015 p.u. to 0.3 p.u. Again the simulation results shown in fig.5 seem to indicate a good transient behaviour with superior performance due to the second approach.

Case 4: Two major perturbations are assumed, i.e., heavy load 1 p.u. and a variation in the inertia constant of the synchronous machine from 10s to 12s. Appreciable performances are obtained for both approaches with a slight edge for the second control law as can be seen in fig.6.

Case 5: Operating point $P_0 = 0.9$ p.u., $Q_0 = 0.3$ p.u. and $x_e = 0.2$ p.u. Again in this case our global fuzzy approaches indicates rapid elimination of oscillations with better performance in the second approach as shown in simulation results are fig.7.

Case 6: Heavy reactive load and weak connection: $P_0 = 0.9$ p.u., $Q_0 = 0.4$ p.u. and $x_e = 0.45$ p.u.

As can be seen from the simulation results shown in Fig.8 both global fuzzy approaches indicates a rapid suppression of oscillations and a better response than the response obtained using conventional stabilizer.

Case 7: Import of reactive power and strong connection: $P_0 = 0.9$ p.u., $Q_0 = -0.4$ p.u. and $x_e = 0.1$ p.u. The simulation results are show in fig.9 clearly indicating a loss of synchronism with CPSS and an effective damping of oscillations with both suggested approaches.
VII. Conclusion

We introduced in this paper, based on the work of Yu & al., a global fuzzy sliding power system stabilizer that enhances damping and thus improves transient dynamics of a single synchronous machine power system. Different load conditions as well as severe perturbations were used to evaluate the proposed global sliding fuzzy power system stabilizer effectiveness in rapidly reducing oscillations that could lead to loss of synchronism if not treated. Simulation results exhibit superior performance over classical PSS and satisfactory transient behaviour for both approaches considered while showing a better performance for the second approach. Multi-machines power system remains to be thoroughly investigated under the proposed technique.

Fig. 10 Speed deviation responses for $P_0 = 0.9p.u$, $Q_0 = 0.3p.u$, and $xe = 0.2p.u$. 

Fig. 11 Speed deviation in heavy reactive load and weak connection case.

Fig. 12 Speed deviation in import of reactive power and strong connection case.

REFERENCES

**Biographies**

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