G-STAR Model for Forecasting Space-Time Variation of Temperature in Northern Ethiopia

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ABSTRACT

Among many indicators of climate change, the temperature is a key indicator to take remedial action for world global warming. This finding provides application of space-time models for temperature data, which is selected in three meteorology stations (Mekelle, Adigrat and Adwa) of Northern Ethiopia. The objectives of this research are to see the space-time variations of temperature and to find better forecasting model. The steps for building this model start from order selection of space and autoregressive order, parameters estimation, a diagnostic check of errors and finally forecasting for the long term. The preliminary model is identified by VAR (vector autoregressive) model and tentatively selects the order by using MIC (minimum information criteria) and uses the autoregressive order for the model and fixes the spatial effect, model parameters are estimated using the least square method. Weighted matrix computed by using queen contiguity criteria. It is found that the model STAR(1,1) and GSTAR(1,1) are two options, finally the best-fitted model is GSTAR(1,1) which has high forecasting performance and smallest RMSEF. The outcome of the forecast indicated that in northern Ethiopia, the weather conditions especially temperature of future is increasing trend in dry seasons in all 3 stations in similar fashion but more consistent and has less variation across the region, and less consistent and high variation within the region and the researcher found that spatial effect has high impact on prediction of models.

1. Introduction

Global warming is the hottest issues of today’s world. Global warming causes droughts, hottest temperature, heavy rainfall, in a short period of time and floods. This problem worries the scientific community, as it could have a major impact on natural and social systems at local, regional and national scales. Climate variability has brought focus to the space-time climatic variability analyses in many regions of the world, especially in the regions that are most prone to the climate change effects. Climate variability studies have got utmost importance during the recent decades. Thanks to the development of statistical techniques to give consistently derived climate estimates at any place at any time. These techniques have successfully been applied in such climatic variability analyses and prediction of the present climate change and future consequence scenario taking both space and time into account.

Ethiopia is one of the sub-Saharan countries located in Eastern Africa. Rapid population growth and deforestation are common throughout the region, rendering it sensitive to changes in climate. This emphasizes the importance of meteorological data and climatic knowledge to the region. In Ethiopia, investigation of long-term variations and
trends in temperature data are not receiving enough attention, even though, the region suffers serious environmental, agricultural and water resource problems. The degree of temperature variability over time is changed not only by natural phenomenon but also by the human activities. Mean annual temperature is often the only index of temperature quoted for a place for the purpose of climate change study. However, the critical question is what is space and time variability and trend of temperature in Ethiopia, and particularly in Northern Ethiopia and what is the result of Ethiopian policy and strategy on reducing greenhouse gas emission to compromise temperature rising and climate changes?

As part of Ethiopia, Northern Ethiopia is also suffering from climatic changes and lacks scientific information about future climatic changes; particularly nothing is known about the space-time temperature variability in the region. No similar research has been conducted in the region. This research mainly intends to develop an appropriate time series model for space and time variability of monthly average temperature at three weather stations in Northern Ethiopia to forecast and to see the performance and handover this model to ENMA (Ethiopian National meteorology agency). The selected multivariate spatial time series model will be used to predict the trend of temperature in the region. It will give statistically supported information about the trend and variation of temperature for those who work on climate change.

2. Literature Review

Many climatologists agree that there has been a large-scale warming of the Earth’s surface over the last hundred years. Some analyses of long time-series of temperatures on the hemispheric and global scale [1] have indicated a warming rate of 0.3-0.6 °C since the mid-19th century, due to anthropogenic causes or astronomic causes [2]. The earth’s surface air temperature is warmed by 0.6±0.2 °C during the 20th century, accompanied by changes in the hydrologic cycle [3]. Of all the climate elements, temperature plays a major role in detecting climate change brought about urbanization and industrialization [4]. The third assessment report projections for the present century are that average temperature rises by 2100 would be in the range of 1.4-5.8 °C [5]. Records show that global temperature averaged worldwide over the land and sea rose 0.6±0.2 °C during the 20th century. On a global scale, climatologically studies indicate an increase of 0.3-0.6 °C of the surface air temperature [3]. When these rates change over time, it can result in profound impacts on our planet. Impact’s like rising sea levels, more extreme weather events, like droughts and floods, melting glaciers, shifts in ecosystems, as well as many others.

In the past, changes in our climate resulted from natural causes, such as differences in the sun’s activity and volcanic eruptions. Greenhouse gases emitted by human activity play a crucial role in warming the Earth’s surface and making it habitable. However, too much human-generated greenhouse gas emissions upset the planet’s natural balance, leading to an increase in warming. Our Earth’s climate change has a direct impact on what we grow and eat in Africa. We know that agriculture is extremely important to Africa’s economy. Seventy percent of the population lives by farming and a third of the income in Africa is generated by agriculture. Most crops in Africa over 95 percent are primarily watered by rainfall. This makes food crops on our continent vulnerable to health stress from our warming planet and extreme weather events linked to climate change. These include changes to seasonal rainfall, droughts and floods. In fact, rain-fed agriculture in Africa could drop by half in 2020 [6].

Climate variability in East Africa is not receiving a lot of attention as the countries are affected by greenhouse gases emitted by developed countries, which are the root causes of the current global climate change. Due to the rise of the greenhouse gas emissions in developed countries, the impacts of climate change will be heavily felt by the countries of East Africa. The impacts of climate change in East Africa include, but are not limited to temperature increase, intense rainfall, a rise in sea-level, and a threat to food security. The increase in Green House Gas emissions will also continue to affect the “natural” climate variability, thus leading to more intense weather events Large variations in temperature are caused by altitude; it is cooler the higher you get. East Africa is a relatively data sparse region of the world. The East African coastal regions are at high risk due to anthropogenic climate change consequences. A range of studies of national climate trends since the 1990s show that mean annual temperatures in Ethiopia have increased by between 0.5 and 1.3 °C. In addition, the frequency of cold nights has decreased significantly in all seasons. Moreover, there is also evidence of a declining trend in rains from 1981-2000 [7]. Given the dependence and reality on rain-fed agriculture in Africa, declining rains could have a negative impact on food security. At the same time, flood events are also reported to be becoming more common, with significant disruptions from flooding occurring in 1997 and 2006 [6].

This concern has motivated the scientific community to conduct research on the space-time temperature variability within the regional scales of the countries. So the researcher interest is in East Africa, the region of East Africa covers
those coastal countries from Eritrea in the north down to Tanzania in the south as well as the Seychelles islands off the coast. While the researchers focus only in Northern Ethiopia and among climate variability indicators like rainfall, drought, deforestation, gas emissions and other factors the researcher focused on temperature recorded on three northern Ethiopian stations.

Objectives;
1. To determine the appropriate space-time model
2. To develop scientific forecasting methods
3. To see the spatial effect of the region in temperature variation and rising
4. To forecasting

3. Research Methods & Models

3.1. Description of Study Area and Data

Ethiopia has 100 meteorological stations over the entire region, out of which seven stations are located in the northern region. Among the seven the researcher focus in three selected stations. The three selected stations are assumed to represent the climatic condition of the region. For the purpose of this study, from Mekelle Meteorology Station uninterrupted temperature series 2006 to 2016 is used, consisting of maximum and minimum air temperatures measured with thermometers in degree Celsius (°C) from each station. The daily readings, maximum and minimum are averaged for the calculation of monthly and used in the study.

3.2. Univariate Time Series: ARIMA Family Models

ARIMA and SARIMA used when the time series displays a regular trend and seasonal variation respectively. It is appropriate to introduce autoregressive and moving average that identify with seasonal and regular trend lags.

\[
\Phi_p(\beta^s)\Phi_p(\beta)^{q} \nabla^p \nabla^q z_t = \theta_Q(\beta)^{\theta}(\beta)
\]

(1)

where \( \Phi_p(\beta^s) = (1 - \Phi_1\beta^s - \cdots - \Phi_p\beta^{sp}) \) is the seasonal AR of order \( p \). \( \Phi_p = (1 - \Phi_1 - \cdots - \Phi_p\beta^p) \) is the regular AR of order \( p \). \( \nabla^p = (1 - \beta)^p \) represents the seasonal difference. \( \nabla^d = (1 - \beta)^d \) is the regular difference. \( \theta_Q(\beta^s) = (1 - \theta_1\beta^s - \cdots - \theta_Q\beta^{Qs}) \) is the seasonal moving average of order \( Q \). \( \theta_Q(\beta) = (1 - \theta_1 - \cdots - \theta_4\beta^4) \) is the regular moving average of order \( q \). \( \varepsilon_t \) is a white noise process [8–10].

3.3. Multivariate Spatial Time Series Models: Generalized Space-Time Autoregressive Model (GSTAR)

Multivariate spatial time series model addressed to the data that depend on time and site or location. It deals with single variable observed over time at a number of different locations. This research application model focused on Space-Time Models.

A classical multivariate time series model VARMA (Vector Autoregressive Moving Average) can be used to analyse space time data, but it requires so many parameters. Cliff and Ord introduced space time model STARMA (space-time autoregressive moving average) and STAR (space-time autoregressive) that have less number of parameters than VARMA model. Then, Pfeifer and Deutsch [11] further studied those models and developed the procedure of their modelling in the STAR model, the autoregressive parameters are assumed to be the same for all locations. This assumption is impractical since different locations usually lead to different parameters. A more flexible model, i.e., the generalized STAR model was proposed by [12], allowing different autoregressive parameters.

GSTAR model is one of the best models that are widely used and applied to predict and model space time data. This model is a modified model of STAR, it was introduced by cliff for the homogenous AR parameters and it developed to GSTAR by [12] for heterogeneous AR parameters across space. For these reasons, the main difference between STAR and GSTAR are only the parameters are the same across all locations (STAR) and differences within each location (GSTAR).

This paper presents the method of STAR and GSTAR modelling through the procedures adopted from Box and Jenkins. The procedures start from making data stationary, parameter estimation and lastly forecasting.
3.4. Space-Time Autoregressive Model (STAR)

STAR Model \((p,p)\) can be written as follows:

\[
Z_t = \sum_{k=1}^{p} \sum_{t=0}^{2k} \phi_{kt} W^l Z_{t-k} + \varepsilon_t
\tag{2}
\]

with

\(Z_t\) : Vector of observation with time \(t\) and location \(n\) and size \(n \times 1\)

\(\phi_{kt}\) : Diagonal matrices of autoregressive parameter order time \(k\) and order space-\(\lambda p\)

\(W^l\) : Weighted matrices which are \(n \times n\)

\(\varepsilon_t\) : Vector of the error term

Example STAR(1,1) for 2 locations; \(Z_t = \phi_{10} Z_{t-1} + \phi_{11} W^l Z_{t-1} + \varepsilon_t\) where \(Z_t = \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix}\), \(Z_{t-1} = \begin{pmatrix} z_{1(t-1)} \\ z_{2(t-1)} \end{pmatrix}\),

\(\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})', w = \begin{pmatrix} 0 & w_{12} \\ w_{21} & 0 \end{pmatrix} \phi_{10} = \begin{pmatrix} \phi_{10}^1 & 0 \\ 0 & \phi_{10}^2 \end{pmatrix} = \phi_{10}^1 = \phi_{10}^2 . \phi_{11} = \begin{pmatrix} \phi_{11}^1 & 0 \\ 0 & \phi_{11}^2 \end{pmatrix} = \phi_{11}^1 = \phi_{11}^2\). Since parameters across locations are homogeneous stationary if \(|\phi_{10} \pm \phi_{11}| < 1\)

3.5. The General Model of GSTAR \((p; \lambda_1 \ldots \lambda_k)\)

\[
Z_t = \sum_{k=1}^{p} \sum_{t=0}^{2k} \phi_{kt} W^l Z_{t-k} + \varepsilon_t
\tag{3}
\]

where \(p\) is the autoregressive order, \(\lambda_k\) is the spatial order of \(k^{th}\) autoregressive term, \(W^l\) is an \(n \times n\) spatial weighted matrix for the spatial order \(l\), \(\phi_{kt}\) is an \(n \times n\) diagonal parameter matrix of temporal lag \(k\) and spatial lag \(l\), \(\varepsilon_t\) is an error vector at time \(t\) which is assumed to be independent and normal distributed with zero mean and constant variance.

Example GSTAR (1, 1) for 2 locations; \(Z_t = \phi_{10} Z_{t-1} + \phi_{11} W^l Z_{t-1} + \varepsilon_t\). Where, \(Z_t = \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix}\), \(Z_{t-1} = \begin{pmatrix} z_{1(t-1)} \\ z_{2(t-1)} \end{pmatrix}\),

\(\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})', w = \begin{pmatrix} 0 & w_{12} \\ w_{21} & 0 \end{pmatrix} \phi_{10} = \begin{pmatrix} \phi_{10}^1 & 0 \\ 0 & \phi_{10}^2 \end{pmatrix}\), and \(\phi_{11} = \begin{pmatrix} \phi_{11}^1 & 0 \\ 0 & \phi_{11}^2 \end{pmatrix}\) stationary if \(|\phi_{10} \pm \phi_{11}| < 1\).

3.6. Identification of Order of Space-Time

Matrix autocorrelation function (MACF)

\[
\hat{\rho}_{ij}(k) = \frac{\sum_{t=1}^{n-k} (x_{it} - \bar{x}_i)(x_{jt+k} - \bar{x}_j)}{\sum_{t=1}^{n} (x_{it} - \bar{x}_i)^2 \sum_{t=1}^{n} (x_{jt} - \bar{x}_j)^2}{\sqrt{2}}
\tag{4}
\]

Matrix partial autocorrelation function (MPACF)

\[
\phi_{kk} = \frac{\text{cov}(Z_t - \bar{Z}_t, Z_{t+k} - \bar{Z}_{t+k})}{\text{var}(Z_t - \bar{Z}_t) \text{var}(Z_{t+k} - \bar{Z}_{t+k})}
\tag{5}
\]

Minimum Akaike information criteria

\[
AIC = \ln(|\Sigma|) + 2r / (T - \frac{r}{K})
\tag{6}
\]

\(|\Sigma|\) : Maximum likelihood estimated from \(\Sigma\)

\(r\) : Number of estimated parameters.

\(T\) : Number of observations.

\(K\) : Number of response variables
Weighted matrix

Contiguity weighted matrix

\[ w_{ij}^* = \frac{w_{ij}}{\sum_{j=1}^{n} w_{ij}}, \quad w_{ij} = \begin{cases} 1, & i \text{ neighbor } j \\ 0, & \text{otherwise} \end{cases} \]

\[
W = \begin{bmatrix}
0 & w_{12} & w_{13} & \cdots & w_{1n} \\
w_{21} & 0 & w_{23} & \cdots & w_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
w_{n1} & w_{n2} & w_{n3} & \cdots & 0
\end{bmatrix}
\]

Inverse weighted matrices: With \( W_{ij} = \frac{1}{d_{ij}} \) and \( d_{ij} = \left\{ \left[ x_i(u_i) - x_j(u_j) \right]^2 + \left[ x_i(v_i) - x_j(v_j) \right] \right\}^{1/2} \). For analysis purpose, the weighted matrix must be changed to row standardized. Queen weighted matrix one value indicated that the sites are neighbours and zero indicates station has no geographically neighbouring [13].

3.7. Parameter Estimation

The general GSTAR model can be written as in equation (7) below

\[ Z_{i(t)} = \sum_{k=1}^{p} \sum_{l=0}^{k} \phi_{kl}^i \left[ w_{11} Z_{1(t-k)} + \cdots + w_{1N} Z_{N(t-N)} \right] + e_{i(t)} \]

The least square estimation method is one of the methods for parameter estimation. Least square estimator of the autoregressive parameter has been derived by [12]. They define new notations. GSTAR(1,1):

\[ Z_{i(t)} = \phi_{10}^i z_{t-1} + \phi_{11}^i w^l z_{t-1} + e_t \]

Model GSTAR(1,1) can be \( Z_i = X_i \beta_i + \epsilon_i \), where,

\[
Z_i = \begin{bmatrix} z_i(1) \\ z_i(2) \\ \vdots \\ z_i(T) \end{bmatrix}_{(T\times1)},
X_i = \begin{bmatrix} z_i(0) \\ v_i(0) \\ \vdots \\ z_i(T-1) \\ v_i(T-1) \end{bmatrix}_{(T\times2)}
\beta_i = \begin{bmatrix} \phi_{10}^{(i)} \\ \phi_{11}^{(i)} \end{bmatrix}_{2\times1},\quad \epsilon_i = \begin{bmatrix} e_i(1) \\ e_i(2) \\ \vdots \\ e_i(T) \end{bmatrix}_{(T\times1)}
\]

where \( v_i(t) = \Sigma_j^N W_{ij} z_j(t) \) for \( t = 0, 1, 2, \ldots, T \) then the overall will be

\[
\begin{bmatrix} z_i(1) \\ z_i(2) \\ \vdots \\ z_i(T) \end{bmatrix}_{(T\times1)} = \begin{bmatrix} z_i(0) \\ \vdots \\ z_i(T-1) \end{bmatrix}_{(T\times1)} + \begin{bmatrix} \phi_{10}^{(i)} \\ \phi_{11}^{(i)} \end{bmatrix}_{2\times1} + \begin{bmatrix} e_i(1) \\ \vdots \\ e_i(T) \end{bmatrix}_{(T\times1)}
\]

This can be

\[ Z_{(NT\times1)} = X_{(NT\times2N)} \beta_{(2N\times1)} + e_{(NT\times1)} \]

\[ \hat{\beta} = \begin{bmatrix} \hat{\phi}_{10}^{(1)} \\ \hat{\phi}_{11}^{(1)} \\ \vdots \\ \hat{\phi}_{10}^{(N)} \\ \hat{\phi}_{11}^{(N)} \end{bmatrix} \]

then by using least square estimation \( \hat{\beta} = ((X'X)^{-1}X'z) \) [14–18].

3.8. Checking Assumptions

In models, residual test (normality, independency (no autocorrelation effect) and homogeneity) must be checked. Normal multivariate testing by looking at the residual plots. White noise test can be done by looking at the plot
MACF and MPACF of the error term. If the plot MACF and MPACF no significant all of its mark as it (.), then an error is independent distributed or has no autocorrelation effect and homogeneity test can be done by MI.

3.9. Forecasting

A one-step forecast for GSTAR (1,1) model is

$$Z_{T+j-1}^l = \phi_1^l Z_{(T+j-1)}^1 + \cdots \phi_{11}^l w^l Z_{(T+j-1)}$$  \hspace{1cm} (9)

For to checking forecast performance, we use mean square error forecast by using data testing and data training to test the performance of forecasting.

Models residual test must be checked. Normal multivariate testing by looking at the residual plots. White noise test can be done by looking at the plot MACF and MPACF of the error term. If the plot MACF and MPACF no significant all of its mark as it (.), then an error is independent distributed or has no autocorrelation effect and homogeneity test can be done.

3.10. Root Mean Square Error Forecast (RMSEF)

$$RMSEF = \sqrt{MSEF} = \sqrt{\frac{1}{M} \sum_{t=1}^{M} (z_t - \hat{z}_t)^2}$$  \hspace{1cm} (10)

where $M$: number of data, $z_t$: Actual data, and $\hat{z}_t$: predicted data.

![Flowchart](image)

**Figure 1.** Method of analysis in the graph
Figure 1 shows that a simple method of analysis in a graphical view, it is easy to understand so, many researchers recommended using such procedure rather than writing a big note in methodology parts.

4. Results & Discussions

As indicated in the methodology part, the data was collected from the meteorology station of Ethiopia in one of its branch Tigrai. The data is from 2006-2016 of 3 Tigrai stations namely Mekele, Adigrat and Adwa. Average of monthly temperature was used for our studies.

Table 1. Summary of descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Z₁</th>
<th>Z₂</th>
<th>Z₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>10.40</td>
<td>10.90</td>
<td>14.20</td>
</tr>
<tr>
<td>1st Qu</td>
<td>16.70</td>
<td>14.88</td>
<td>18.27</td>
</tr>
<tr>
<td>Median</td>
<td>17.90</td>
<td>15.90</td>
<td>19.40</td>
</tr>
<tr>
<td>Mean</td>
<td>17.98</td>
<td>15.98</td>
<td>19.73</td>
</tr>
<tr>
<td>3rd Qu</td>
<td>19.20</td>
<td>17.02</td>
<td>21.20</td>
</tr>
<tr>
<td>Max.</td>
<td>21.90</td>
<td>27.10</td>
<td>25.00</td>
</tr>
</tbody>
</table>

In Table 1 as we see, we can observe that in Station 2 recorded the max average temperature. In Station 2, 25% of the observation lies above 17-degree centigrade, while in station one and 2 is 14 & 15 respectively. On the other side, the 3 stations have a maximum recorded average temperature in Celsius is from 21 to 27 and the minimum is from 10 to 15. For the collected data of the three stations informally to check the stationary see in the appendix; the researchers used plot after regular difference and formally the Dickey-Fuller test since the STAR or GSTAR model needs to be stationary. Hence,

\( H₀: \) Data not Stationary \( H₁: \) Data stationary

Table 2. Dickey-Fuller unit root tests results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Rho</th>
<th>Pr &lt; Rho</th>
<th>Tau</th>
<th>Pr &lt; Rho</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z₁</td>
<td>-56.76</td>
<td>0.0004</td>
<td>-5.18</td>
<td>0.0002</td>
</tr>
<tr>
<td>Z₂</td>
<td>-45.26</td>
<td>0.0004</td>
<td>-4.66</td>
<td>0.0013</td>
</tr>
<tr>
<td>Z₃</td>
<td>-84.79</td>
<td>0.0004</td>
<td>-6.39</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

For Table 2 the DF test shows that the data is stationary since Dickey-Fuller Unit Root Tests p-value is less than the alpha level of significance which indicates that the data is stationary. Moreover, the informal test or plot shows the same thing. If the given data is not stationary other researcher need to apply more difference (either seasonal or annual) or need to use box-cox transformation method.

4.1. Identification of the Order of Space and Time

It is the most important point to come up the true GSTAR or STAR Model, still there is no exact and simple method to find this model however the simple method is to use the order of Vector Autoregressive (VAR) model and fix the spatial to one. Here are the selection criteria of the order of VAR Model by using mpacf, macf and mic, in this research the researcher applied MIC as follows:

Table 3. MIC results

<table>
<thead>
<tr>
<th>Lag</th>
<th>MA 1</th>
<th>MA 2</th>
<th>MA 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 1</td>
<td>1.7702267</td>
<td>1.8576453</td>
<td>1.9275282</td>
</tr>
<tr>
<td>AR 2</td>
<td>1.8510589</td>
<td>1.9470721</td>
<td>2.0341701</td>
</tr>
<tr>
<td>AR 3</td>
<td>1.9300222</td>
<td>1.9975903</td>
<td>2.0629736</td>
</tr>
</tbody>
</table>

From Table 3 we can say that the order of the VAR Model is (1,1) since the MACF is cut off after the first lag and the MPACF also almost cut off after first lag and also the MIC value pointed that the order is (1,1) since 1.77 is the smallest value in the given value of MIC which indicated that the order of the VAR is (1,1). Now to come up the STAR and GSTAR Model since we know the order the next step will be finding the weighted matrices and parameter estimation of the STAR and GSTAR Model. Remember the weighted matrices are used queen contiguity now the
selected study areas are three regions which are in northern Ethiopia (Mekelle, Adigrat and Adwa of Tigray Region) they are geographically connected, so the row standardized weights matrices is as follows:

### Table 4. Standardized weighted matrix

<table>
<thead>
<tr>
<th>Locations (Station)</th>
<th>South Tigray (Mekelle)</th>
<th>East Tigray (Adigrat)</th>
<th>Central Tigray (Adwa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST (Me)</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>ET (Ag)</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>CT (Aw)</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
</tr>
</tbody>
</table>

In Table 4, the row standardized matrices are each observation divided by the sum of weights for features neighbours. This will add in our model the spatial effect of neighbouring on climate and environmental changes. Now we need to write the model of STAR and GSTAR and estimate the parameters. Remember the basic difference between the two models are the parameters are homogenous and heterogeneous across locations respectively as shown in equations below:

**STAR Model:** Let be \( \varphi^{1}_{10} = a, \varphi^{1}_{11} = b, \varphi^{1*}_{11} = c \)

\[
Z_{1t} = \varphi^{1}_{10}Z_{1t-1} + W_{21} \varphi^{1}_{11}Z_{2t-1} + W_{31} \varphi^{1*}_{11}Z_{3t-1} \\
Z_{2t} = \varphi^{1}_{20}Z_{2t-1} + W_{12} \varphi^{1}_{11}Z_{1t-1} + W_{22} \varphi^{1*}_{11}Z_{3t-1} \\
Z_{3t} = \varphi^{1}_{30}Z_{3t-1} + W_{13} \varphi^{1}_{11}Z_{1t-1} + W_{23} \varphi^{1*}_{11}Z_{2t-1}
\]

(11)

**GSTAR Model:** Let be \( \varphi^{2}_{10} = a, \varphi^{2}_{11} = b, \varphi^{2*}_{11} = c \)

\[
\varphi^{2}_{10} = g, \varphi^{2}_{11} = h, \varphi^{2*}_{11} = i \\
Z_{1t} = \varphi^{2}_{10}Z_{1t-1} + W_{21} \varphi^{2}_{11}Z_{2t-1} + W_{31} \varphi^{2*}_{11}Z_{3t-1} \\
Z_{2t} = \varphi^{2}_{20}Z_{2t-1} + W_{12} \varphi^{2}_{11}Z_{1t-1} + W_{22} \varphi^{2*}_{11}Z_{3t-1} \\
Z_{3t} = \varphi^{2}_{30}Z_{3t-1} + W_{13} \varphi^{2}_{11}Z_{1t-1} + W_{23} \varphi^{2*}_{11}Z_{2t-1}
\]

(12)

### Table 5. STAR parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Err.</th>
<th>t Value</th>
<th>Pr &gt;</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.860178</td>
<td>0.0355</td>
<td>24.23</td>
<td>.0001</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.195780</td>
<td>0.0848</td>
<td>2.31</td>
<td>0.0225</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0.082256</td>
<td>0.0550</td>
<td>1.49</td>
<td>0.1374</td>
<td></td>
</tr>
</tbody>
</table>

From Table 5, the STAR models are as follows:

\[
Z_1 = 0.8602 * lag_1Z + 0.5 * 0.1958 * lag_1Z + 0.5 * 0.0823 * lag_1Z \\
= 0.860lag_1Z + 0.098lag_1Z + 0.041lag_1Z \\
Z_2 = 0.195 * lag_2Z + 0.5 * 0.8602 * lag_1Z + 0.5 * 0.0823 * lag_1Z \\
= 0.196lag_1Z + 0.430lag_1Z + 0.041lag_1Z \\
Z_3 = 0.0823 * lag_3Z + 0.5 * 0.8602 * lag_1Z + 0.5 * 0.1958 * lag_1Z \\
= 0.082lag_1Z + 0.430lag_1Z + 0.098lag_1Z
\]

(13)

From Table 6, GSTAR models are as follows:

\[
Z_1 = 0.8602 * lag_1Z + 0.5 * 0.1958 * lag_1Z + 0.5 * 0.0823 * lag_1Z \\
= 0.490lag_1Z + 0.191lag_1Z + 0.308lag_1Z \\
Z_2 = 0.3188 * lag_1Z + 0.5 * 0.5991 * lag_1Z + 0.5 * 0.5545 * lag_1Z \\
= 0.320lag_1Z + 0.300lag_1Z + 0.277lag_1Z \\
Z_3 = 0.8347 * lag_3Z + 0.5 * 0.1144 * lag_1Z + 0.5 * 0.2707 * lag_1Z \\
= 0.835lag_1Z + 0.057lag_1Z + 0.135lag_1Z
\]

(14)
The above result shows that parameter estimation results and final models. Based on the above models, our aim is to forecast but before forecasting, it is better to see the assumptions of the error term. If the MPCF and MAPCF have no effect on the error term and also there is no problem of autocorrelation plus error term shows multivariate normal so our model is good enough. MACF of the Error term and MPACF of the Error term show that all error lies b/n the given standard deviation so it shows our model is good. So from this, the error term has no significant effect on our model plus the normal plot shows that it normal and single test of autocorrelation also shows it the error is independent. To forecast let's see the performance and compare the best model from STAR(1,1) and GSTAR(1,1) the details as follows:

Table 6. GSTAR Parameters estimates

| Parameter | Estimate | Std. Err. | t Value | Pr > |t|
|-----------|----------|-----------|---------|------|
| a1        | 0.489485 | 0.0936    | 5.23    | <.0001|
| b1        | 0.382532 | 0.1563    | 2.45    | 0.0157|
| c1        | 0.615670 | 0.1452    | 4.24    | <.0001|
| a2        | 0.318854 | 0.0963    | 3.31    | 0.0012|
| b2        | 0.599176 | 0.2306    | 2.60    | 0.0105|
| c2        | 0.554560 | 0.1790    | 3.10    | 0.0024|
| a3        | 0.834708 | 0.1013    | 8.24    | <.0001|
| b3        | 0.114456 | 0.2611    | 0.44    | 0.6618|
| c3        | 0.270728 | 0.2180    | 1.24    | 0.2167|

From Table 7, we can say that the forecasting performance of GSTAR is better than the STAR model. So we use the GSTAR Model to forecast the long term. See the result in the appendix. As we expected the result of STAR model forecasting gives less accurate value forecast for the three places this shows some weakness of STAR model so it is not vital to include the result beside that GSTAR works well the reason is the parameters are different across the model. Then the GSTAR forecast as follows:

Table 7. RMSEF results

<table>
<thead>
<tr>
<th>MODEL</th>
<th>RMSEF</th>
</tr>
</thead>
<tbody>
<tr>
<td>STAR(1,1)</td>
<td>0.139</td>
</tr>
<tr>
<td>GSTAR(1,1)</td>
<td>0.100</td>
</tr>
</tbody>
</table>

Table 8. GSTAR forecasting

<table>
<thead>
<tr>
<th>Year</th>
<th>Location</th>
<th>J</th>
<th>F</th>
<th>M</th>
<th>A</th>
<th>J</th>
<th>J</th>
<th>O</th>
<th>S</th>
<th>O</th>
<th>N</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>2017</td>
<td>Z1</td>
<td>18.26</td>
<td>19.30</td>
<td>19.79</td>
<td>20.90</td>
<td>19.81</td>
<td>17.49</td>
<td>17.70</td>
<td>17.28</td>
<td>16.68</td>
<td>16.12</td>
<td>15.10</td>
</tr>
<tr>
<td></td>
<td>Z2</td>
<td>16.42</td>
<td>17.37</td>
<td>17.75</td>
<td>18.60</td>
<td>17.57</td>
<td>15.56</td>
<td>15.80</td>
<td>15.39</td>
<td>14.94</td>
<td>14.39</td>
<td>13.55</td>
</tr>
<tr>
<td></td>
<td>Z3</td>
<td>18.77</td>
<td>19.63</td>
<td>20.16</td>
<td>21.65</td>
<td>20.93</td>
<td>18.32</td>
<td>18.38</td>
<td>18.01</td>
<td>17.20</td>
<td>16.59</td>
<td>15.65</td>
</tr>
<tr>
<td></td>
<td>Z2</td>
<td>15.08</td>
<td>16.65</td>
<td>17.77</td>
<td>18.28</td>
<td>17.80</td>
<td>16.43</td>
<td>16.43</td>
<td>16.41</td>
<td>15.73</td>
<td>14.68</td>
<td>14.13</td>
</tr>
<tr>
<td>2019</td>
<td>Z1</td>
<td>18.25</td>
<td>19.60</td>
<td>20.03</td>
<td>20.81</td>
<td>22.10</td>
<td>18.19</td>
<td>18.29</td>
<td>18.43</td>
<td>17.78</td>
<td>16.79</td>
<td>16.44</td>
</tr>
<tr>
<td></td>
<td>Z2</td>
<td>16.42</td>
<td>17.61</td>
<td>17.96</td>
<td>18.66</td>
<td>19.73</td>
<td>16.29</td>
<td>16.41</td>
<td>16.47</td>
<td>15.94</td>
<td>15.08</td>
<td>14.66</td>
</tr>
<tr>
<td>2020</td>
<td>Z1</td>
<td>17.94</td>
<td>18.54</td>
<td>20.59</td>
<td>20.84</td>
<td>20.17</td>
<td>17.59</td>
<td>16.96</td>
<td>16.70</td>
<td>16.26</td>
<td>15.23</td>
<td>14.49</td>
</tr>
</tbody>
</table>

GSTAR model forecast of the 3 neighbouring cities (Z1-Mekele, Z2-Adigrat and Z3-Adwa) shows that there is up trend in the first dry season and down in others, it also shows that the spatial effect is really vital for forecasting climate dynamics as shown here in both cities the highest temperature event occurs in May and coolest temperature occurs in November.
5. Results & Discussions

Modelling and forecasting space-time can be done by using many methods but here we prove that the generalized space-time methods are very crucial since the parameters across space are different and it increases the accuracy of the forecast as we seen in RMSEF. However, Vector Autoregressive (VAR) does not use weighted matrix which is important to determine the distance or the neighbourhood of the place effect plus it assumes no criteria on the parameters, Space-Time Autoregressive (STAR) is better than VAR model however it assumes the parameters across the space is the same which reflected two or more station explain by the same parameters which lead to bigger RMSEF and the result of the forecast less accurate. So in this research, the researcher focused on GSTAR model since it has better forecasting performance. Forecasting became one of the goals of modelling multivariate time series data and the accuracy of forecasting can be measured using Root Mean Square Error Forecast (RMSEF), hence the small value of RMSEF indicates good forecasting performance of the model.

Each model has a value of different forecasting accuracy, depending on the characteristics of the order of time at each Location/Stations. Hence in our model GSTAR (1, 1) better fit for the given data and forecasted for long period of time and the temperature of future is up in the dry season and down in others but more consistent and has less variation across the region.

Recommendation to EPM: The researcher recommended to Ethiopian policymakers (EPM), in EGTP 2 (Ethiopian growth and transformation plan II) must give the same attention to environmental protection as poverty reduction.

Recommendations to ENMA: Ethiopian national Meteorology agency is the only organization which forecasts whether the condition in Ethiopia, so based on this finding for better forecasting GSTAR model has more accuracy and it is appropriate to be used in future.

Recommendation to Researchers: The limitation of this study is not considering seasonal effects and does not consider informal statistical tests (using formal test is also become strength for statistician but sometimes formal test is difficult to understand for non-statistician reader) so, other scientist can be used the steps of this model but includes seasonal effect models, informal, formal tests and other variables in their study to get better finding and policy briefs.

Reference


