



Fixed Points of Expansive Maps in Partial Cone Metric Spaces

Jerolina FERNANDEZ^{1,*}, Kalpana SAXENA², Neeraj MALVIYA³

^{1,3}*Department of Mathematics, NRI Institute of Information Science and Technology, Bhopal, M.P., India*

²*Department of Mathematics, Govt. M.V.M College, Bhopal, M.P., India*

Received: 03/06/2014 Revised: 07/08/2014 Accepted: 12/08/2014

ABSTRACT

Matthews [10] introduced a new distance P on a nonempty set X , which he called a partial metric. Sonmez [16] introduced the notion of partial cone metric space and its topological characterization. In present paper, we define expansive map in partial cone metric space and prove some fixed point theorems for such maps. Our results extend the results of [4, 9, and 17] in partial cone metric space. An example is given to support the usability of our results.

Keywords: Partial cone metric space, expanding mapping, fixed point

2010 AMS Subject Classification: 54H25, 47H10

1. INTRODUCTION

In 1994, Matthews [10] introduced the notion of partial metric space as a part of the study of denotational semantics of dataflow networks. He generalized the concept of metric space in the sense that the distance from a point to itself need not be equal to zero. In 1984, Wang et.al [17] introduced the concept of expanding mappings and proved some fixed point theorems in complete metric spaces.

In 1980, Rzepecki [15] introduced a generalized metric d_E on a set X in a way that $d_E: X \times X \rightarrow P$, replacing the set of real numbers with a Banach space E in the metric function where P is a normal cone in E with a partial order \leq .

Seven years later, Lin [7] considered the notion of cone metric spaces by replacing real numbers with a cone P in the metric function in which it is called a K -metric. Twenty years after Lin's work, Long-Guang and Xian [3] announced the notion of a cone metric space by replacing real numbers with an ordering Banach space. The authors discussed some properties of convergence of sequences and proved some fixed point theorems in cone metric spaces showing that metric spaces really does not provide enough space for the fixed point theory. Indeed, they gave an example of a cone metric space (X, d) and proved existence of a unique fixed point for a self map T of X which is contractive in the category of cone metric spaces but is not contractive in the category of metric spaces.

*Corresponding author, e-mail: jerolinafernandez@gmail.com

After that, cone metric spaces have been studied by many other authors (see [1], [12], [13]).

Very recently, in 2013 A.Sonmez [16] introduced the notion of partial cone metric space and its topological characterization. The author developed some fixed point theorems in this generalized setting. On the other hand, the research about fixed points of expansive mapping was initiated by Machuca (see [8]). Later Jungck discussed fixed points for other forms of expansive mapping (see [6]). In 1982, Wang et al. (see [17]) presented some interesting work on expansive mappings in metric spaces

which correspond to some contractive mapping in [14]. Also, Zhang has done considerable work in this field. In order to generalize the results about fixed point theory, Zhang (See [18]) published his work Fixed Point Theory and Its Applications, in which the fixed point problem for expansive mapping is systematically presented in a chapter.

In the present paper, we define expansive map and present some fixed point results for these maps in partial cone metric space and conclude with an example.

2. PRELIMINARIES

First, we invite some standard notations and definitions in cone metric spaces and partial cone metric spaces.

A cone P is a subset of a real Banach space E such that

- (i) P is closed, nonempty and $P \neq \{0\}$;
- (ii) if a, b are nonnegative real numbers and $x, y \in P$, then $ax+by \in P$;
- (iii) $P \cap (-P) = \{0\}$.

Given a cone $P \subseteq E$, we define a partial ordering \leq with respect to P by $x \leq y$ if and only if $y-x \in P$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y-x \in \text{int}P$, $\text{int}P$ denotes the interior of P .

The cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \text{ implies } \|x\| \leq K \|y\|.$$

The least positive number satisfying above is called the normal constant of P .

The cone P is called regular if every increasing sequence which is bounded from above is convergent. That is, if $\{x_n\}$ is sequence such that $x_1 \leq x_2 \leq \dots \leq x_n \leq \dots \leq y$ for some $y \in E$, then there is $x \in E$ such that $\|x_n - x\| \rightarrow 0 (n \rightarrow \infty)$. Equivalently, the cone P is regular if and only if every decreasing sequence which is bounded from below is convergent. It is well known that a regular cone is a normal cone.

Definition 2.1. [3] Let X be a nonempty set. Suppose the mapping $d: X \times X \rightarrow E$ satisfies

- (d1) $0 < d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;
- (d2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
- (d3) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X and (X, d) is called a cone metric space.

Definition 2.2. [16] A partial cone metric on a nonempty set X is a function $p: X \times X \rightarrow E$ such that for all $x, y, z \in X$:

- (p1) $x=y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,
- (p2) $0 \leq p(x,x) \leq p(x,y)$,
- (p3) $p(x,y) = p(y,x)$,
- (p4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial cone metric space is a pair (X, p) such that X is a nonempty set and p is a partial cone metric on X . It is clear that, if $p(x, y) = 0$, then from (p₁) and (p₂) $x=y$. But if $x=y$, $p(x, y)$ may not be 0.

A cone metric space is a partial cone metric space. But there are partial cone metric spaces which are not cone metric spaces. The following an example illustrate a partial cone metric space but not a cone metric space.

Example 2.3. [16] Let $E = R^2$, $P = \{(x, y) \in E : x, y \geq 0\}$, $X = R^+$ and $p : X \times X \rightarrow E$ is defined by

$$p(x, y) = (\max\{x, y\}, \alpha \max\{x, y\})$$

where $\alpha \geq 0$ is a constant. Then (X, p) is a partial cone metric space which is not a cone metric space.

Theorem 2.4. [16] Any partial cone metric space (X, p) is a topological space.

Theorem 2.5. [16] Let (X, p) be a partial cone metric space and P be a normal cone with normal constant K , then (X, p) is T_0 .

Definition 2.6. [16] Let (X, p) be a partial cone metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in \text{int}P$, there is N such that for all $n > N$, $p(x_n, x) \ll c + p(x, x)$, then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x , and x is the limit of $\{x_n\}$. We denote this by $\lim_{n \rightarrow \infty} x_n = x$ or, $x_n \rightarrow x (n \rightarrow \infty)$.

Theorem 2.7. [16] Let (X, p) be a partial cone metric space, P be a normal cone with normal constant K . Let $\{x_n\}$ be a sequence in X . Then $\{x_n\}$ converges to x if and only if $p(x_n, x) \rightarrow p(x, x) (n \rightarrow \infty)$.

Sonmez [16] also noted that if (X, p) is a partial cone metric space, P be a normal cone with normal constant K and $p(x_n, x) \rightarrow p(x, x) (n \rightarrow \infty)$, then $p(x_n, x_n) \rightarrow p(x, x) (n \rightarrow \infty)$.

Lemma 2.8. [16] Let $\{x_n\}$ be a sequence in partial cone metric space (X, p) . If a point x is the limit of $\{x_n\}$ and $p(y, y) = p(y, x)$ then y is the limit point of $\{x_n\}$.

Definition 2.9. [16] Let (X, p) be a partial cone metric space. $\{x_n\}$ be a sequence in X . $\{x_n\}$ is Cauchy sequence if there is a $\epsilon \in P$ such that for every $\epsilon > 0$ there is N such that for all $n, m > N$

$$\|p(x_n, x_m) - a\| < \epsilon.$$

Definition 2.10. [16] A partial cone metric space (X, p) is said to be complete if every Cauchy sequence in (X, p) is convergent in (X, p) .

Theorem 2.11. [16] Let (X, p) be a partial cone metric space. If $\{x_n\}$ is a Cauchy sequence in (X, p) , then it is a Cauchy sequence in the cone metric space (X, d) .

Proposition 2.12[5]: Let (X, d) be a cone metric space and P be a cone in a real Banach space E . If $u \leq v, v \ll w$ then $u \ll w$.

Proposition 2.13[11]: Let P be a cone and $0 \leq u \ll c$ for each $c \in \text{int}P$, then $u = 0$.

Proposition 2.14[2]: Let P be a cone in a real Banach space E . If $a \in P$ and $a \leq ka$, for some $k \in [0, 1)$ then $a = 0$.

3. Expansive map: In this section, we define expansive map in partial cone metric space

Definition 3.1: Let (X, p) be a partial cone metric space. A map $f: X \rightarrow X$ is said to be an expansive mapping if there exists a constant $k > 1$ such that $p(fx, fy) \geq k p(x, y)$ for all $x, y \in X$.

Example 3.2: Let (X, p) be a partial cone metric space as defined in example 2.3. Define a self map $f: X \rightarrow X$ by $fx = \beta x$ where $\beta > 1$, for all $x \in X$. Clearly f is an expansive map in X .

4. MAIN RESULTS

In this section, we will present some fixed point theorems for expanding mappings in the partial cone metric spaces. Furthermore, we conclude with an example.

Theorem 4.1: Let (X, p) be a complete partial cone metric space with respect to a cone P contained in a real Banach space E . Let f and g be two surjective self map of X satisfying

$$p(fx, gy) + k[p(x, gy) + p(y, fx)] \geq a p(x, fx) + b p(y, gy) + c p(x, y) \dots (4.1.1)$$

for every $x, y \in X, x \neq y$ where $a, b, c, k \geq 0$ and $a+b+c > 1+2k$ and $c > 2k$. Then f and g have a unique fixed point in X .

Proof: We define a sequence $\{x_n\}$ as follows for $n = 0, 1, 2, 3, \dots$

$$x_{2n} = fx_{2n+1}, x_{2n+1} = gx_{2n+2}, \dots \dots \dots (4.1.2)$$

If $x_{2n} = x_{2n+1} = x_{2n+2}$, for some n then we see that x_{2n} is a fixed point of f and g .

Therefore, we suppose that no two consecutive terms of sequence $\{x_n\}$ are equal.

Now, we put $x = x_{2n+1}$ and $y = x_{2n+2}$ in (4.1.1)

$$\begin{aligned} p(fx_{2n+1}, gx_{2n+2}) + k[p(x_{2n+1}, gx_{2n+2}) + p(x_{2n+2}, fx_{2n+1})] &\geq a p(x_{2n+1}, fx_{2n+1}) + b p(x_{2n+2}, gx_{2n+2}) + c p(x_{2n+1}, x_{2n+2}) \\ \Rightarrow p(x_{2n}, x_{2n+1}) + k[p(x_{2n+1}, x_{2n+1}) + p(x_{2n+2}, x_{2n})] &\geq a p(x_{2n+1}, x_{2n}) + b p(x_{2n+2}, x_{2n+1}) + c p(x_{2n+1}, x_{2n+2}) \\ \Rightarrow p(x_{2n}, x_{2n+1}) + k[p(x_{2n+1}, x_{2n+1}) + p(x_{2n+2}, x_{2n+1}) + p(x_{2n}, x_{2n+1}) - p(x_{2n+1}, x_{2n+1})] &\geq a p(x_{2n+1}, x_{2n}) + b p(x_{2n+2}, x_{2n+1}) + c p(x_{2n+1}, x_{2n+2}) \\ \Rightarrow p(x_{2n}, x_{2n+1}) + k[p(x_{2n+2}, x_{2n+1}) + p(x_{2n+1}, x_{2n})] &\geq a p(x_{2n+1}, x_{2n}) + b p(x_{2n+2}, x_{2n+1}) + c p(x_{2n+1}, x_{2n+2}) \\ \Rightarrow (1 + k - a) p(x_{2n}, x_{2n+1}) &\geq (b + c - k) p(x_{2n+1}, x_{2n+2}) \\ \Rightarrow p(x_{2n+1}, x_{2n+2}) &\leq \frac{(1 + k - a) p(x_{2n}, x_{2n+1})}{(b + c - k)} \\ \Rightarrow p(x_{2n+1}, x_{2n+2}) &\leq k_1 p(x_{2n}, x_{2n+1}) \quad \text{where } k_1 = \frac{(1 + k - a)}{(b + c - k)} < 1 \quad (\text{As } a + b + c > 1 + 2k) \end{aligned}$$

Similarly, we can calculate

$$p(x_{2n+2}, x_{2n+3}) \leq k_2 p(x_{2n+1}, x_{2n+2}) \text{ where } k_2 = \frac{(1 + k - b)}{(a + c - k)} < 1. (\text{As } a + b + c > 1 + 2k) \text{ and so on.}$$

So, in general

$$p(x_n, x_{n+1}) \leq k p(x_{n-1}, x_n) \text{ for } n = 1, 2, 3, 4, \dots$$

where $k = \max \{k_1, k_2\}$ then $k < 1$

$$\Rightarrow p(x_n, x_{n+1}) \leq k^n p(x_{n-1}, x_n)$$

Now, we shall prove that $\{x_n\}$ is a Cauchy sequence. For $m > n$,

$$\begin{aligned} p(x_n, x_m) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + \dots + p(x_{m-1}, x_m) - \sum_{r=1}^{m-n-1} p(x_{m+r}, x_{m+r}) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + \dots + p(x_{m-1}, x_m) \\ &\leq (k^n + k^{n+1} + \dots + k^{m-1}) p(x_1, x_0) \\ &\leq (1 + k + \dots + k^{m-n-1}) k^n p(x_1, x_0) \\ &< \frac{k^n}{1-k} p(x_1, x_0) \end{aligned}$$

$$\text{And } \| p(x_n, x_m) \| \leq \frac{k^n}{1-k} K \| p(x_1, x_0) \|$$

This implies that $p(x_m, x_n) \rightarrow 0$ (as $n, m \rightarrow \infty$). Since $\frac{k^n}{1-k} K \| p(x_1, x_0) \| \rightarrow 0$ as $n \rightarrow \infty$.

Therefore, $\{x_n\}$ is a Cauchy sequence. By completeness of X , there is $x^* \in X$, such that $x_n \rightarrow x^*$ (as $n \rightarrow \infty$).

$$\text{Therefore, } \lim_{n \rightarrow \infty} p(x_n, x^*) = p(x^*, x^*) = \lim_{n \rightarrow \infty} p(x_n, x_n) = 0.$$

Existence of fixed point: Since f and g are two surjective self maps in X and hence there exist two points y and y' in X such that

$$\begin{aligned} x^* = fy \text{ and } x^* = gy' \dots \dots \dots (4.1.3) \\ p(x_{2n}, x^*) = p(fx_{2n+1}, gy') \\ \geq -k [p(x_{2n+1}, gy') + p(y', fx_{2n+1})] + a p(x_{2n+1}, fx_{2n+1}) + b p(y', gy') + c p(x_{2n+1}, y') \\ \geq -k [p(x_{2n+1}, gy') + p(y', x_{2n})] + a p(x_{2n+1}, x_{2n}) + b p(y', gy') + c p(x_{2n+1}, y') \end{aligned}$$

As $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are subsequences of $\{x_n\}$ as $n \rightarrow \infty$, $\{x_{2n}\} \rightarrow x^*$, $\{x_{2n+1}\} \rightarrow x^*$

Therefore, $p(x^*, x^*) \geq -k [p(x^*, x^*) + p(y', x^*)] + a p(x^*, x^*) + b p(y', x^*) + c p(x^*, y')$
 $\frac{(b+c-k)}{(1-a+k)} \|p(x^*, y')\| \leq \|p(x^*, x^*)\|$ [Using p_3]
 $\Rightarrow p(x^*, y') = 0$ [as $\frac{(b+c-k)}{(1-a+k)} > 1$, i.e. $a + b + c > 1 + 2k$ and $p(x^*, x^*) = 0$]
 $\Rightarrow x^* = y'$(4.1.4)

In the similar way, we can prove that $x^* = y$(4.1.5)

The fact that (4.1.3) along with (4.1.4) and (4.1.5) shows that x^* is a common fixed point of f and g .

Uniqueness

Let z be another common fixed point of f and g , that is

$fz = z$ and $gz = z$ (4.1.6)

then $p(x^*, z) = p(fx^*, gz)$

$\geq -k [p(x^*, gz) + p(z, fx^*)] + a p(x^*, fx^*) + b p(z, gz) + c p(x^*, z)$
 $\Rightarrow p(x^*, z) \geq -k [p(x^*, z) + p(z, x^*)] + a p(x^*, x^*) + b p(z, z) + c p(x^*, z)$
 $= (-2k+c)p(x^*, z)$ [Using p_3]
 $\Rightarrow p(x^*, z) \leq \frac{1}{(c-2k)} p(x^*, z)$

$\Rightarrow p(x^*, z) = 0$ (As $c > 2k$ and by Prop. 2.13)
 $\Rightarrow x^* = z$

This completes the proof of the theorem 4.1. □

Theorem 4.2. Let (X, p) be a complete partial cone metric space and $T: X \rightarrow X$ be a continuous surjection. Suppose that there exist a constant $\lambda > 1$ such that, for each $x, y \in X$,

$p(Tx, Ty) \geq \lambda u$, for some $u \in \{p(x, y), p(x, Tx), p(y, Ty)\}$ (4.2.1)

Then T has a fixed point in X .

Proof: We can obtain a sequence $\{x_n\}$ such that $x_{n-1} = Tx_n$.

Without loss of generality, we assume that $x_{n-1} \neq x_n$ for all $n = 1, 2, \dots$.
 (Otherwise, if there exists some n_0 such that $x_{n_0-1} = x_{n_0}$, then x_{n_0} is a fixed point of T).
 It follows that from condition (4.2.1)

$p(x_{n-1}, x_n) = p(Tx_n, Tx_{n+1}) \geq \lambda u_n$
 where $u_n = \{p(x_n, x_{n+1}), p(x_n, x_{n-1})\}$.

Now we have to consider the following two cases.

If $u_n = p(x_n, x_{n-1})$, then
 $p(x_{n-1}, x_n) \geq \lambda p(x_n, x_{n-1})$ (4.2.2)
 which implies $p(x_{n-1}, x_n) = 0$ (by Prop. 2.13), that is $x_{n-1} = x_n$. This is a contradiction.

If $u_n = p(x_n, x_{n+1})$, then
 $p(x_{n-1}, x_n) \geq \lambda p(x_n, x_{n+1})$(4.2.3)
 $\Rightarrow p(x_n, x_{n+1}) \leq \frac{1}{\lambda} p(x_{n-1}, x_n)$
 $\Rightarrow p(x_n, x_{n+1}) \leq k p(x_{n-1}, x_n)$ [where $k = \frac{1}{\lambda} < 1$]

We can prove $\{x_n\}$ is a Cauchy sequence in X using (4.2.3) as proved in theorem 4.1. Since (X, p) is complete, the sequence $\{x_n\}$ converges to a point $z \in X$. Since T is continuous, it is clear that z is a fixed point of T . This completes the proof of the

theorem 4.2. □

Corollary 4.3. Let (X, p) be a complete partial cone metric space with respect to a cone P contained in a real Banach space E . Let f and g be two surjective self maps of X satisfying

$$p(fx, gy) \geq c p(x, y) \dots\dots\dots(4.3.1)$$

where $c > 1$. Then f and g have a unique fixed point in X .

Proof: If we put $k, a, b = 0$ in theorem 4.1, then we get above corollary 4.3.

Corollary 4.4. Let (X, p) be a complete partial cone metric space with respect to a cone P contained in a real Banach space E . Let f be a surjective self map of X satisfying

$$p(fx, fy) \geq c p(x, y) \dots\dots\dots(4.4.1)$$

Where $c > 1$. Then f has a unique fixed point in X .

Proof: If we put $f = g$ in corollary 4.3, then we get above corollary 4.4 which is an extension of theorem 1 of Wang et al. [17] in partial cone metric space.

Corollary 4.5. Let (X, p) be a partial cone metric space and $f: X \rightarrow X$ be a surjection. Suppose that there exist a positive integer n and a real number $c > 1$ such that $p(f^n x, f^n y) \geq c p(x, y)$ for all $x, y \in X$. Then f has a unique fixed point in X .

Proof: From corollary 4.4, f^n has a unique fixed point z . But $f^n(fz) = f(f^n z) = fz$, so fz is also a fixed point of f^n . Hence $fz = z$, z is a fixed point of f . Since the fixed point of f is also fixed point of f^n , the fixed point of f is unique.

Corollary 4.6. Let (X, p) be a complete partial cone metric space with respect to a cone P contained in a real Banach space E . Let f and g be two surjective self maps of X satisfying

$$p(fx, gy) \geq a p(x, fx) + b p(y, gy) + c p(x, y) \dots\dots\dots(4.6.1)$$

for every $x, y \in X, x \neq y$ where $a, b, c \geq 0$ and $c > 1$. Then f and g have a unique fixed point in X .

Proof: The proof is similar to proof of the theorem 4.1.

Corollary 4.7. Let (X, p) be a complete partial cone metric space with respect to a cone P contained in a real Banach space E . Let f be a surjective self map of X satisfying

$$p(fx, fy) \geq a p(x, fx) + b p(y, fy) + c p(x, y) \dots\dots\dots(4.7.1)$$

for every $x, y \in X, x \neq y$ where $a, b, c \geq 0$ and $c > 1$. Then f has a unique fixed point in X .

Proof: If we put $f = g$ in corollary 4.6, then we get above corollary 4.7 which is an extension of theorem 2 of Wang et al. [17] in partial cone metric space.

The following example demonstrates corollary 4.3.

Example 4.8. Let (X, p) be a partial cone metric space as defined in example 2.3. Define a self map f on X as follows $fx = 2x$ for all $x \in X$. Clearly f is an expansive mapping. If we take $c = 2$ then condition (4.4.1) holds trivially good and 0 is the unique fixed point of the map f .

Remark 1: If mappings are continuous in theorem 4.1 then existence of fixed point follows very easily. As shown below

$$x = \lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} f x_{2n+1} = f \lim_{n \rightarrow \infty} x_{2n+1} = fx \text{ (as } n \rightarrow \infty, \{x_{2n+1}\} \rightarrow \infty)$$

$$\text{Similarly, } x = \lim_{n \rightarrow \infty} x_{2n+1} = \lim_{n \rightarrow \infty} g x_{2n+2} = g \lim_{n \rightarrow \infty} x_{2n+2} = gx \text{ (as } n \rightarrow \infty, \{x_{2n+2}\} \rightarrow \infty)$$

Remark 2: In corollary 4.6, we proved the fixed point is unique by using only $c > 1$ and there is no need of $a < 1$ and $b < 1$, so it extend and unify the theorem 2 of Wang et al.[17] in partial cone metric space.

5. CONCLUSION

In this paper, we defined expansive maps in new structure (i.e partial cone metric space) and prove various fixed point theorems for expansive maps which are the extension of well known famous results [4], [9], [17]. An example is also given to support the theorem.

ACKNOWLEDGEMENT

The first author is thankful to Prof. Geeta Modi [Govt. Motilal Vigyan Mahavidhyalaya, Bhopal (M.P.) India] for constant encouragement.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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