ON SOME COMBINATORIAL IDENTITIES INVOLVING THE TERMS OF GENERALIZED FIBONACCI AND LUCAS SEQUENCES

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Abstract
In this paper, we consider the Horadam sequence and some summation formulas involving the terms of the Horadam sequence. We derive combinatorial identities by using the trace, the determinant, and the $n$th power of a special matrix.

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1. Preliminaries

Generalized Fibonacci sequence $W_n = W_n(a, b; p, q)$ is defined as follows;

(1.1) $W_n = pW_{n-1} - qW_{n-2}$, $W_0 = a$, $W_1 = b$.

Where $a$, $b$, $p$, and $q$ are arbitrary complex numbers, with $q \neq 0$. Since, these numbers have been studied firstly by Horadam(see, e.g., [1]) they are called as Horadam numbers. Some special cases of this sequence such as

(1.2) $U_n = W_n(0, 1; p, q)$, $V_n = W_n(2, p; p, q)$

were investigated by Lucas[6]. Further and in detailed knowledge can be found in[1, 2, 3, 4, 5, 6]. If $\alpha, \beta$ assumed distinct, are the roots of

(1.3) $\lambda^2 - p\lambda + q = 0$

then the sequence $W_n$ has the Binet representation

(1.4) $W_n = \frac{A\alpha^n - B\beta^n}{\alpha - \beta}$,