

# ANTI-INVARIANT $\xi^\perp$ -RIEMANNIAN SUBMERSIONS FROM ALMOST CONTACT MANIFOLDS

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## Abstract

We introduce anti-invariant  $\xi^\perp$ -Riemannian submersions from almost contact manifolds onto Riemannian manifolds. We give an example, investigate the geometry of foliations which are arisen from the definition of a Riemannian submersion and check the harmonicity of such submersions. We also find necessary and sufficient conditions for a special anti-invariant  $\xi^\perp$ -Riemannian submersion to be totally geodesic. Moreover, we obtain decomposition theorems for the total manifold of such submersions.

**Keywords:** Riemannian submersion, Sasakian manifold, Anti-invariant  $\xi^\perp$ -Riemannian submersion

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## 1. Introduction

Riemannian submersions between Riemannian manifolds were studied by O'Neil [9] and Gray [7]. In [13], Waston defined almost Hermitian submersions between almost Hermitian manifolds and he showed that the base manifold and each fiber has the same kind of structure as the total space, in most cases. He also showed that the vertical and horizontal distributions are invariant. On the other hand, the geometry of anti-invariant Riemannian submersions is quite different from the geometry of almost Hermitian submersions. For example, since every holomorphic map between Kähler manifolds is harmonic [5], it follows that any holomorphic submersion between Kähler manifolds is harmonic. However, this result is not valid for anti-invariant Riemannian submersions, which was first studied by Sahin in [11]. Similarly, Ianus and Pastore [8] shows  $\phi$ -holomorphic maps between contact manifolds are harmonic. This implies that

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