Hacettepe Journal of Mathematics and Statistics
Volume 42 (3) (2013), 269 – 280

OPERATOR VALUED DIRICHLET PROBLEM IN THE PLANE

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Received 26 : 04 : 2012 : Accepted 21 : 06 : 2012

Abstract

We consider an operator valued Dirichlet problem for harmonic mappings and prove the existence of a Perron-like solution. To formulate the Perron's construction we make use of Olson's notion of spectral order. We introduce a class of operator valued subharmonic mappings and establish some of their elementary properties.

Keywords: Operator theory, Harmonic mappings, Perron method, Spectral order.
2000 AMS Classification: 47A56, 47A63, 47B15

1. Introduction

With this paper we would like to initiate a research on potential theory of harmonic and subharmonic functions in the plane with values in the class of bounded linear operators on a Hilbert space. The main purpose is to describe the solution of the Dirichlet problem using a Perron's method. Making use of the Olson's notion of spectral order we show that there is a Perron-like solution whenever the boundary values are commuting or the Poisson integral of the boundary mapping is a projection (Theorem 4.10).

We recall that inequality in spectral order implies the inequality in the usual order on the class of self-adjoint operators. By Olson $\mathcal{S}$ is a conditionally complete lattice under the partial order $\preceq$. Making use of functional calculus and spectral order we obtain some generalizations of well known properties of subharmonic functions to the operator valued case. One of the main results of the paper is the maximum principle for subharmonic mappings in terms of the spectral order.

Recently quite a few papers are written about harmonic mappings which take their values in infinite dimensional spaces. We consider in this paper a Dirichlet problem for operator valued harmonic mappings of the complex plane. To formulate the Perron solution we introduce a class of operator valued subharmonic mappings and establish some of their elementary properties which will be used for our purpose. In section 4 we provide an example of a mapping of the form $F(z) = T + zS$, where T and S are

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