Hacettepe Journal of Mathematics and Statistics
Volume 42 (3) (2013), 289 – 297

NEW INTEGRAL INEQUALITIES VIA (α, m)-CONVEXITY AND QUASI-CONVEXITY

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Received 29 : 04 : 2012 : Accepted 19 : 10 : 2012

Abstract

In this paper, we establish some new integral inequalities involving Beta function via (α, m)-convexity and quasi-convexity, respectively. Our results in special cases recapture known results.

Keywords: Hermite's inequality, Euler Beta function, Hölder's inequality, (α, m)-convexity, quasi-convexity

2000 AMS Classification: 26D15, 33B15, 26A51, 39B62.

1. Introduction

Let I be an interval in $\mathbb{R}$. Then $f : I \to \mathbb{R}$ is said to be convex (see [17, P.1]) if

$$f\left(tx + (1-t)\, y\right) \leq tf\left(x\right) + (1-t)\, f\left(y\right)$$

holds for all $x, y \in I$ and $t \in [0, 1]$.

In [27], Toader defined m-convexity as follows:

1.1. Definition. The function $f : [0, b] \to \mathbb{R}$, $b > 0$ is said to be m-convex, where $m \in [0, 1]$, if

$$f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y)$$

holds for all $x, y \in [0, b]$ and $t \in [0, 1]$.We say that f is $m-$concave if $-f$ is $m-$convex.

In [18], Miheşan defined $(\alpha, m)-$ convexity as follows:

1.2. Definition. The function $f : [0, b] \to \mathbb{R}$, $b > 0$, is said to be $(\alpha, m)-$ convex, where $(\alpha, m) \in [0, 1]^2$, if

$$f(tx + m(1-t)y) \leq t^{\alpha} f(x) + m(1-t^{\alpha})f(y)$$

holds for all $x, y \in [0, b]$ and $t \in [0, 1]$.

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