Mobile Robot Localization via Outlier Rejection in Sonar Range Sensor Data

Sezcan YILMAZ¹, Hilal EZERCAN KAYIR², Burak KALECI², Osman PARLAKTUNA²

Eskisehir Osmangazi University, ¹Department of Mechanical Engineering and ²Department of Electrical Engineering, 26480, Eskisehir, Turkey
Tel: +90-222-239-3750, Fax: +90-222-239-3613
{sezcan, burakaleci}@gmail.com, {hezercan, oparlak}@ogu.edu.tr

Abstract: Localization is an important ability for a mobile robot. The probabilistic localization methods become more popular because of the ability of representing the uncertainties of the sensor measurements and inaccuracies in environments. They also provide robust solutions for different localization problems. The particle filter is one of the probabilistic localization methods. In this study, sonar range sensors are used for mobile robot localization. Sonar range sensors suffer from wrong reflections that may result outliers in the data set. Outliers may also occur in the particle filter process. In this study, a new sensor model Repealing Range Sensor Model (R²SM) is proposed and integrated to particle filter to reduce the effects of outliers. In order to show the effectiveness of the proposed method, experiments are conducted and the results are compared with a well-known outlier rejection method, Grubbs’ T-Test. Experiments show that results of the proposed approach are comparable to the results of the Grubbs’ T-Test in terms of Localization Success Ratio (LSR) and Number of Iterations (NOI) required for localization. The main advantage of the proposed R²SM is that it does not require any additional information such as critical value table. This provides more flexible outlier rejection approach.

Keywords: Particle Filter, Localization, Sonar Range Sensor, Outlier Rejection.

1. Introduction

Robot localization is one of the important topics in the mobile robotics research society. The process of estimating robot configuration (position and orientation) related to a given map of the environment could be defined as the localization problem. The probabilistic approaches are one of the most popular and commonly used methods among the localization solutions. They provide useful representation to describe uncertainties related to sensor and environment in the estimation process. A known probabilistic approach is Kalman Filter [1], [2]. The Kalman filter provides estimation of a posterior distribution of robot configuration by using odometer and range sensors. However, the Kalman filter has an important limitation that the initial configuration of the robot must be known. In order to cope with the limitation of the Kalman filter, Bayesian-based localization methods have been studied. Particle Filter is a Bayesian-based localization method and it has been commonly known as Monte Carlo Localization (MCL) and was introduced by Dellaert [3] and Fox [4]. In MCL, randomly drawn particles are used instead of describing a probability density function.

One of the important issues of probabilistic localization methods is how raw sensor measurements are converted to localization information. For this purpose, the observation (or sensor) model, \( p(y|x, m) \) which is defined as the probability of measurement \( y \) with respect to robot’s configuration \( x \) and map information \( m \), is used. Several sensor models for particle filter were presented in the literature [5], [6]. These studies have specific parameters and do not provide a reliable adaptation for different density functions. In [7] and [8], the authors consider the characteristic of the likelihood function and they observe that if the function is peaked, the number of samples required for successful localization increase. In order to overcome this problem, a different sensor model is proposed [6].

Mobile robots usually use sonar, laser range finder and camera measurements in sensor model [3], [4]. Laser range finder provides sensitive angular resolution and accurate readings. On the other hand, sonar range finder is mostly used in mobile robots...
2. Background

2.1. Bayes Filtering

Bayes filter estimates the state (configuration) $x$ of a robot in an environment by using sensor measurements. Bayesian approaches assume that the environment is Markovian, that is the past and future measurements are independent from the current ones [15].

Assume that $x_t$ is the state vector of robot configuration $\{x_t = [x_r, y_r, \theta_r]^T\}$ at time $t$, where $x_r$, $y_r$ are the position and $\theta_r$ is the orientation components. Let $u_t$ be the action vector of robot and $y_t$ be the sensor readings at time $t$. The main idea behind the Bayes filters is to estimate the posterior density by using measurements. Generally, the posterior is named belief and defined as follows:

$$Bel(x_t) = p(x_t|u_t, y_t)$$ (1)

The initial belief describes the initial value of the state. In the global localization, the robot has no information about its state. Therefore, a uniform distribution is used for the initial belief.

Bayes filters estimate the belief of the robot by using two recursive steps: Prediction and update steps. In the first step, the motion model is used to integrate the movements to the current posterior. The motion model is described as conditional density $p(x_t|x_{t-1}, u_t)$. The predictive density over $x_t$ is as follows:

$$p(x_t|u_{t-1}, y_{t-1}) = \int p(x_{t-1}|x_{t-1}, u_{t-1}) p(x_{t-1}|u_{t-1}, y_{t-1}) dx_{t-1}$$ (2)

In the second step the sensor model is used. The sensor model is expressed in terms of likelihood $p(y_t|x_t)$ and is described as the likelihood to be at $x_t$ with the sensor measurements $y_t$. The resulting posterior density over $x_t$ is as follows:

$$p(x_t|u_t, y_t) = \frac{p(y_t|x_t) p(x_t|u_{t-1}, y_{t-1})}{p(y_t|u_{t-1}, y_{t-1})}$$ (3)

2.2. Particle Filter

Particle Filter represents the belief by a set of $N$ weighted samples.

$$Bel(x_t) = \{x_t^i, w_t^i, i = 1, \ldots, N\}$$ (4)

where $x_t^i$ represent the state and $w_t^i$ the importance factor of the $i^{th}$ sample at time $t$. In global localization,
initially all particles have the same importance factor, that is $1/N$ [16].

In analogy with the Bayes filter, the particle filter estimates the belief of the samples by using two recursive steps. In the first step, the motion model is applied to all particles and the predictive density $\hat{x}_t^i, i = 1, ... N$ is obtained as in equation 5.

$$\hat{x}_t^i = p(x_t^i | x_{t-1}^i, u_{t-1})$$

(5)

Then, the sensor model is applied to the predictive density to calculate the importance factor of all particles.

$$w_t^i = p(y_t | \hat{x}_t^i)$$

(6)

The new sample set is obtained from the predictive density $\hat{x}_t^i$ according to the importance factor of the samples $w_t^i$.

### 2.3. Grubbs’ T-Test

An outlier can be defined as the data in a given data set that does not belong to the same characteristic with the rest of the data. For example, most of the data could be close to a linear line while the outliers may lie far away from the close neighborhood of the line. Also, an outlier is an extreme data in a distribution. An outlier example is shown in Fig. 1.

![Outlier example](image)

**Figure 1.** Outlier example

The outlier data may cause undesired effects in decision making process. In order to avoid the effects of the outliers, one can remove the outlier data from the data set. In this stage, the potential outliers should be examined carefully because they may result from an inherent error such as calculation, sensing, etc. or they correctly describe an extreme situation and the data should be taken into account in decision making. Therefore the outlier detection is an important issue. In literature, there are several outlier detection (rejection) methods. Grubbs’ T-Test is one of the most known outlier rejection methods. It is appropriate for normally distributed data sets and has an easy procedure as follows:

**Step1:** Calculate $T$ value that represents the distance of a point from the others:

$$T = \frac{|p - \bar{y}|}{\sigma}$$

(7)

where $p$ is a point in the set, $\bar{y}$ and $\sigma$ are the mean and the standard deviation of the data set, respectively.

**Step2:** Grubbs’ T-Test has a critical value table that includes threshold values to determine the outlier data. Generally, the rows and the columns of the table show the number of data $n$, and the number of potential outliers that you would encounter $\alpha$, respectively [17]. If $T$ is greater than $\alpha$, the data is accepted as outlier and rejected from the data set.

### 3. Proposed Method

#### 3.2. Problem Definition

Particle filter-based mobile robot localization method that uses sonar range sensors suffers two important drawbacks. One of them is caused by the nature of the sonar range sensor. Sonar range sensors have an important disadvantage that can be called as wrong reflections. In Fig 2, mirror-like reflections, high-order reflections or cross-talk are given, respectively.

![Outlier example](image)

(a) (b) (c)

**Figure 2.** a) Mirror-like reflection. b) High-order reflection. c) Cross-talk.

The other drawback is caused by the particle filter process. In the traditional particle filter [3], the total sensor probability is calculated by multiplication of individual sensor probabilities. The total sensor probability represents the importance factor for each particle. As a result, the importance factor for a particle is calculated as follows:

$$w_t^i = \prod_{k=1}^{n} p(y_t^k | \hat{x}_t^i)$$

(8)

where $n$ represents the number of sensors.
In some cases, some of the probabilities might be much different than the expected values and the total sensor probability is negatively affected. Mathematically, these cases can be expressed as:

\[ p(y_t^k|x_t^k) > p(y_t^k|x_t^l) \]
\[ \text{or} \]
\[ p(y_t^k|x_t^k) < p(y_t^k|x_t^l) \]  \hspace{1cm} (9)

where

\[ \bar{y}_t^k = E(y_t^k) \]  \hspace{1cm} (10)

Two examples about this phenomenon are given in Fig. 3-b and 3-c. In Fig. 3-a, the robot is shown at the correct configuration and the lines indicate the distance measured by the sensors. In Fig. 3-b and 3-c, two particles and their assumed distance measurements are shown. The particle in Fig. 3-b is in the neighborhood of the correct robot configuration and the sensor reading shown with an arrow is much different than the expected reading. Thus, the probability of this reading becomes much smaller than the probability of other readings and the total sensor probability is dominated by this low sensor probability. As a result, the particle that is supposed to survive is negatively affected and it may be eliminated. Therefore, this adverse probability case could be named as survival case. Fig. 3-c shows another case which could be called removal case. Here, the particle is in a different configuration than the correct configuration of the robot. However, the sensor readings given with arrows are approximately equal to the actual readings. Therefore, these sensor readings will have higher probabilities than the rest of the sensor readings and cause high total sensor probability. In this situation, the particle may survive although it is placed in a wrong configuration. The cases mentioned above can be named as adverse probability effects.

In this paper, a new sensor model is proposed in order to localize the robot by using sonar range sensors. The proposed method is named Repealing Range Sensor Model (R^2SM). It is capable of detecting and rejecting the outlier that is caused by both the particle filter and sonar range sensor.

### 3.3. Repealing Range Sensor Model

In this study, a new sensor model, Repealing Range Sensor Model (R^2SM), is proposed in order to eliminate the effects of the adverse probabilities. In this subsection, firstly, two illustrative examples about survival and removal adverse probability cases are introduced. In order to explain the procedure of the proposed method these two examples are used. Then, detailed algorithm of the R^2SM is given.

#### Case 1: Survival Case

In Fig. 4, the sensor probabilities for the particle that is in survival case are shown. In the figure, individual sensor probabilities are represented with bars, the total sensor probability and the arithmetic mean of the sensor probabilities are given with red dashed line and red solid line, respectively.

The sensor probabilities which are calculated by using Eq. 6 for a survival case-particle are given in Fig. 4-a. R^2SM, firstly, determines the mean of the sensor probabilities and the absolute deviation of each individual sensor from the mean are calculated. The probabilities are sorted in descending absolute deviation order and are shown in Fig. 4-b. After that, R^2SM decides the leader which is defined as the highest-deviation sensor probability. The leader is always placed at the first column of the Fig. 4-b. In this case, the leader is below the mean. The probability values are considered as outliers until the next probability value is greater than the mean. The outliers are shown with black bars in Fig. 4-b. Then, outliers are rejected and the total sensor probability is recalculated with the remaining sensor probabilities. As a result, the total sensor probability that is calculated without outliers is higher than the total sensor probability that is calculated with outliers. Thus, the particle that would be eliminated if the R^2SM is not used survives when R^2SM is used.

#### Case 2: Removal Case

The sensor probabilities for a particle in the removal case are represented in Fig. 5. R^2SM procedure is almost the same for the survival case. However, in the removal case, the leader is above the mean. Therefore, the probability values that are met until the probability value that is less than the mean are determined as outlier. Fig. 5-b shows the outliers with black bars. Consequently, the total sensor probability decreases by the use of R^2SM. Thus, the particle that would be survived if the R^2SM is not used is removed when R^2SM is used.
Figure 4. a) Sensor probabilities. b) Sensor probabilities sorted with respect to deviation from the mean.

Figure 5. a) Sensor probabilities. b) Sensor probabilities sorted with respect to deviation from the mean.
R²SM receives individual sensor probabilities \( p(y_k^i|x_t^i) \) for \( i^{th} \) particle at the time \( t \) where \( k = 1, \ldots, n \) and \( n \) is number of sensors that are used as inputs. Additionally, an empty list \( Q \) is used to hold the sensor probabilities and related information about them. The total sensor probability \( (w_t^i) \) of the \( i^{th} \) particle at the time \( t \) is resulted from the R²SM as output. If \( N \) represents the number of particles, R²SM algorithm repeats \( N \) times and localization algorithm runs \( O(nxN) \). The details of the algorithm are given as follows:

**Input:** Sensor probabilities \( p(y_k^i|x_t^i) \) and an empty list \( Q \).

**Output:** Total sensor probability \( (w_t^i) \).

**Step 1:** Calculate the mean of the sensor probabilities.

\[
\mu_t^i = \frac{\sum_{k=1}^{n} p(y_k^i|x_t^i)}{n}
\]  

(11)

**Step 2:** For each individual probability, calculate the absolute deviation from the mean.

\[
d_t^{ik} = |\mu_t^i - p(y_k^i|x_t^i)|, \quad k = 1, \ldots, n
\]  

(12)

**Step 3:** For each individual probability, determine the sign parameter.

\[
s_t^{ik} = \begin{cases} 
1 & p(y_k^i|x_t^i) \geq \mu_t^i \\
-1 & p(y_k^i|x_t^i) < \mu_t^i
\end{cases}
\]  

(13)

**Step 4:** For each individual probability, insert the sensor probabilities, absolute deviations, and sign parameters into \( Q \) list.

\[
Q \leftarrow Q + \{p(y_k^i|x_t^i), d_t^{ik}, s_t^{ik}\}, \quad k = 1, \ldots, n
\]  

(14)

**Step 5:** Sort the \( Q \) list in descending order with respect to absolute deviations.

**Step 6:** Specify the first item of the \( Q \) list as the leader. Store the sign parameter of the leader in \( s_t^{i,1} \).

\[
s_t^{i,1} \leftarrow s_t^{i,1}
\]  

(15)

**Step 7:** Remove the first item of \( Q \) from the list and update the number of sensor probabilities.

\[
Q \leftarrow Q - \{p(y_k^i|x_t^i), d_t^{i,1}, s_t^{i,1}\} \\
n \leftarrow n - 1
\]  

(16)

(17)

**Step 8:** Control the first item of the \( Q \) list

- If \( s_t^{i,1} \times s_t^{i,1} = 1 \), return to Step 7.
- If \( s_t^{i,1} \times s_t^{i,1} = -1 \), go to Step 9.

**Step 9:** Calculate total sensor probability and return \( (w_t^i) \).

\[
w_t^i = \left(\prod_{k=1}^{n} p(y_k^i|x_t^i)\right)^{1/n}
\]  

(18)

The R²SM provides that the particles in the neighborhood of the correct robot configuration may have high sensor probability. On the other hand, the particles at different configurations than the correct one may have low sensor probability. Therefore, the R²SM forces the particles to concentrate around the actual robot configuration in fewer steps than the traditional sensor model. As a result, the R²SM algorithm improves the localization success and decreases duration of localization than the case with the traditional sensor model.

### 4. Application and Analysis of the Proposed Method

In this section, the proposed Particle filter approach is applied to localize a Pioneer P3-DX robot in a laboratory environment. The P3-DX has a balanced drive system which includes two-wheel differential drive, caster wheel, and high-resolution motion encoders. It has also wireless Ethernet networking system and Pentium-based onboard computer system [18]. The sensors on the robot are: 16 ultrasonic sensors, a SICK LMS200 laser range finder, a PTZ Camera, and a compass. The sonar range finder is used for the applications.

![Figure 6. A Pioneer P3-DX robot in the laboratory environment](image)

The applications were realized in the Eskişehir Osmangazi University Electric-Electronic Engineering Department Artificial Intelligence and Robotics Laboratory (Fig. 6). The width and height of the experiment environment are 7300mm and 8500mm, respectively. The map of the experimental environment and the path followed by the robot at localization process are shown in Fig. 7. Data from compass, 16 sonar, 180 laser range finder data, the position coordinates, and orientation angle are recorded into a txt file at every 1000 msec. Later, the
txt file is used as the input of the proposed localization method.

![Figure 7. The environment and path used in the experiments](image)

**Analysis of the Proposed Method**

In order to analyze the results of the proposed approach, first some definitions are given:

- NOS (Number of Samples): Density of the samples in Unit Sample Space (USS).
- NOI (Number of Iteration): Number of iterations of the system that successful localization is achieved.
- LSR (Localization Success Ratio): Ratio of the number of successful localizations and total number of experiments.

In this study, the USS for the position and orientation are chosen as $1m^2$ and $180^\circ$, respectively. Results of $R^2$SM, Grubbs’ T-Test, and no outlier rejection methods are compared in terms of NOI and LSR for 40 NOS and different number of sonar sensors. It is important to note that the comparison for NOI is done by using only successful experiments. Additionally, in the experiments, the start point to the localization process is randomly determined.

In order to localize mobile robot correctly and quickly, the accurate and sufficient data must be injected into the localization process. It is expected that NOI increases due to deleting data when the outlier rejection methods (Grubbs’ T-Test and $R^2$SM) are applied. Although some data are deleted, the characteristic of NOI with respect to number of sensor remains same. The reason of this result is that outlier rejection methods remove only disruptive data. Therefore, the outliers are not taken into account on the localization process and NOI does not affected. The results are shown in Fig. 8.

In particle filter without outlier rejection methods, the localization success is negatively affected from the adverse probability conditions. However, the outlier rejection methods eliminate the adverse-probability sensor readings. Thus, the LSR is clearly improved. As shown in Fig. 9, the results for $R^2$SM and Grubbs’ T-Test are similar. The advantage of the $R^2$SM is that it does not require any additional information such as critical value table.

![Figure 8. NOI versus Number of Sensor](image)

![Figure 9. LSR versus Number of Sensor](image)

**5. Conclusions**

In this study, a particle filter-based localization method for mobile robot is proposed. Sonar range sensors are used for localization. It is clear that, sonar sensors have many drawbacks that affect the localization performance. Generally, the localization with sonar sensors are more complicated than other sensors. In order to cope with this trouble, a new sensor model $R^2$SM is integrated to the particle filter. $R^2$SM acts as an outlier rejection method and eliminate the disruptive sensor data. The performance of the proposed method is compared with well-known outlier rejection method Grubbs’ T-Test. Both methods are implemented and examined in the experimental environment. The results show that both outlier rejection methods have similar performance in terms of NOI and LSR. The proposed $R^2$SM is more preferable than other statistical outlier rejection methods because $R^2$SM does not need any additional parameters such as critical value tables.
6. References


Sezcan YILMAZ received the BA and MS degree in Mechanical Engineering from the Eskisehir Osmangazi University (ESOGU), Eskisehir, Turkey. He has been working at the Department of Mechanical Engineering of the ESOGU since 2004. Currently, he is pursuing the PhD degree in Mechanical Engineering. His research interests include haptic devices and mobile robotics.

Hilal EZERCAN KAYIR is a PhD student of Electrical–Electronics Engineering at Eskisehir Osmangazi University, Turkey. She earned the BS degree in Electrical – Electronics Engineering from Dokuz Eylul University and MS degree from Pamukkale University, Turkey. Her research interests include mobile robotics and control.

Burak KALECI is a PhD student at the Eskisehir Osmangazi University, Turkey. He received the BS and MS degree in Electrical and Electronics Engineering from the Eskisehir Osmangazi University, Turkey. His research interests includes mobile robot localization, and multi-robot coordination systems.

Osman PARLAKTUNA is an Professor of Electrical–Electronics Engineering at Eskisehir Osmangazi University, Turkey. He earned the BS and MS Degrees in Electrical Engineering from the Middle East Technical University, Ankara, Turkey, and the PhD from Vanderbilt University, Nashville, TN. His research interests include mobile robotics, automation and control.