ANALYSIS OF FILTER CHARACTERISTICS BASED ON PWL MEMRISTOR

Şuayb Çağrı YENER1, Reşat MUTLU2, Hulusi Hakan KUNTMAN3
1 Sakarya University, Engineering Faculty, Electrical and Electronics Engineering Department 54187, Sakarya, TURKEY
2 Namık Kemal University, Çorlu Engineering Faculty, Electronics and Communication Engineering Department, Çorlu, Tekirdağ, TURKEY
3 Istanbul Technical University, Electrical – Electronics Faculty, Electronics and Communication Engineering Department 34469, Maslak, İstanbul, TURKEY
syener@sakarya.edu.tr, rmutlu@nku.edu.tr, kuntman@itu.edu.tr

Abstract: Memristor is claimed as a passive two terminal fundamental circuit element in 1971 and declared to be physically fabricated in 2008. The memristor can provide new features in analog circuit design thanks to its properties which cannot be mimicked by a resistor, a capacitor and an inductor. It is believed that in future it can be used in analog applications such as programmable amplifiers, oscillators, filters and chaotic sources. In this paper, detailed simulations of a low-pass filter and a high-pass filter with memristor are done using a piece-wise linear hypothetical memristor characteristic for sinusoidal input. The memristive switching issue in these filters is analyzed. Also the total harmonic distortions of the memristor based filters are inspected as a function of charge and the input signal frequency using simulations.

Keywords: Memristor, analog filter design, memristor-based filter, PWL model and total harmonic distortion

1. Introduction

Leon Chua has theoretically predicted that there must be one more fundamental circuit element besides the three basic circuit elements, resistor, capacitor and inductor. Chua named the new element memristor and laid out its properties in 1971 [1]. Chua and Kang described “memristive devices and systems” which are systems with similar properties to memristors in 1976 [2]. Nearly four decades later than its prediction, in 2008, a group of researchers from HP laboratories announced their discovery of the missing circuit element memristor and have given a physical model for it [3]. Its thin-film realization has resulted in interest on memristor continuing in an increasing rate.

It is expected that, memristors can be applied and provide new additional features to analog circuits. Analog applications such as programmable gain amplifiers, programmable oscillators, Schmitt-trigger circuits and threshold comparators using memristors have been already studied in literature [4-7].

Variable resistors provide essential features for analog circuit design. Memristor also has variable resistance and this functionality can provide many new properties while defining and adjusting the characteristics of the circuits Application of the memristor to analog filters can bring out many new features thanks to its variable memristance. There are already some studies in the literature related to application of the memristors to analog filters [8-16].

In this paper, a comprehensive study is done on memristor based memristor-capacitor (M-C) low-pass (LP) and high-pass (HP) filters. Some previously unmentioned properties of memristor
based filters such as Total Harmonic Distortion (THD) considering Memristive Switching (MS) are analyzed. For the first time in the literature to the best of our knowledge, we will demonstrate the effect of memristive switching and THD of memristor on waveforms of the LP or HP filters.

Organization of the paper is as follows. The second section briefly explains memristor element and also gives its generalized piece-wise linear (PWL) modeling. The third section introduces memristor based low-pass and HP filters. Also dynamic model of M-C filters is presented in this section. Simulation results are presented in fourth section. Gain, and THD analyses are done. Performance characteristics of the filters are simulated and results obtained have been presented. The paper is concluded with results, considerations and notices on memristive filter design.

2. Definition and Fundamentals of Memristor

Memristor maintains a functional relationship between the time integrals of the current and the voltage. The terminal equation of the charge-controlled memristor is expressed as

\[ v = M(q)i \]  

(1)

Memristance function of the charge-controlled memristor is given as

\[ M(q) = \frac{d\phi(q)}{dq} \]  

(2)

Where \( \phi \) is the memristor flux and \( q \) is the memristor charge. They are described as

\[ q(t) = \int_{-\infty}^{t} i(\tau)d\tau \]  

(3)

\[ \phi(t) = \int_{-\infty}^{t} v(\tau)d\tau \]  

(4)

In this paper, flux-charge characteristics and memristance function are used to model memristor.

3. PWL Modeling of Memristor

The PWL memristor characteristic has been used first by Chua in 1971 [1]. The memristor flux is a monotonically-increasing function with respect to memristor charge. Previously, piece-wise linear memristor models have been used in several studies [17-22]. A PWL flux-charge characteristic is also used in this paper for analysis of LP and HP filters. Beyond the basically used two-segment piece-wise model, a generic PWL flux-charge characteristic with \( n \) segment is shown in Figure 1.

\[ \phi = Mq \]

(5)

where \( k \) is the segment number. Also the memristance of the \( k \)th segment, \( M_k \), is the slope of \( k \)th segment in Figure 1 and expressed by,

\[ M(q) = \frac{d\phi(q)}{dq} = \{M_k \ , q_k < q < q_{k+1}\} \]  

(6)

Variation of memristance as a function of memristor charge for the PWL characteristic given in Figure 1 is illustrated in Figure 2.
It is obvious that the slope of the curve and the memristance is constant throughout the same segment. Thus, the memristor operating on the same segment acts as a constant resistor. However, values of the parameters such as amplitude of input voltage $V_m$, initial charge $q_0$, etc. may change the operating regime so the memristance value may not remain constant and the memristor operates in different segments or takes different memristance values. This sudden jump of memristance of a memristor is called as “memristive switching” or “resistive switching”.

Corresponding zero-crossing I-V hysteresis loops are shown in Figure 3 for no memristive switching and memristive switching situations, respectively. Due to the piece-wise linear nature of the memristor characteristics, the I-V hysteresis loop have also different segments if MS occurs as shown in Figure 3 (b).

4. Memristor Based Filters

Analog filters are used commonly in electronic circuits. Passive first-order RC filters with one resistor and one capacitor are also called one-pole filters. While HP filters pass high frequency signals and blocks low-frequency signals, LP filters behave in opposite characterization.

Basic R-C LP and HP filters are shown in Figure 4 (a) and (c), respectively. By replacing the linear resistor with memristor, M-C form of these filters are obtained as illustrated in Figure 4 (b) and (d) for LP and HP, respectively. In the following sections, properties and characteristics of memristor-based filters are inspected.

4.0.1. Memristor Based Low-Pass Filter When There is No Memristive Switching

If memristor memristance is constant, memristive switching does not occur. Then, a transfer function can be used to describe this LP filter circuit as in R-C LP filters. The transfer function of such a memristor-based LP filter and corresponding amplitude and phase expressions are defined by

$$H(j\omega) = \frac{1}{1 + j\omega M \tau C} = \frac{1}{1 + j\omega/\omega_c}$$  \hspace{1cm} (7)

$$A(\omega) = |H(j\omega)| = \frac{1}{\sqrt{1 + (\omega M \tau C)^2}}$$  \hspace{1cm} (8)
4.0.2. Memristor Based High-Pass Filter When There is No Memristive Switching

Similar to M-C LP filter, while memristance is not varying, the corresponding transfer function, amplitude and phase of the memristor-based HP filter as in R-C HP filters are defined by

\[
H(j\omega) = \frac{j\omega \mathcal{M}_C}{1 + j\omega \mathcal{M}_C} = \frac{j \omega / \omega_C}{1 + j \omega / \omega_C} \quad (10)
\]

\[
A(\omega) = |H(j\omega)| = \frac{\omega \mathcal{M}_C}{\sqrt{1 + (\omega \mathcal{M}_C)^2}} \quad (11)
\]

\[
\phi(\omega) = \angle H(j\omega) = \tan^{-1}\left(\frac{\omega_C}{\omega}\right) \quad (12)
\]

Since both M-C LP and HP filters act as an R-C filter when memristor memristance is constant or when there is no resistive switching, the amplitude and phase of M-C filter is not inspected further.

4.1. Low-Pass and High-Pass Filter Behavior When There is No Memristive Switching

In this study, a five-segment symmetric PWL memristance function given in (13) is used for both of the filter topologies.

\[
M(q) = \begin{cases} 
M_1, & q_1 < q < q_2, \\
M_2, & q_2 < q < q_3, \\
M_3, & q_3 < q < -q_1, \\
M_4, & q_1 < q < -q_2, \\
M_5, & q < -q_2, \\
M_6, & q < -q_3 \end{cases} \quad (13)
\]

4.2. Dynamic Model of M-C Filters

If memristive switching occurs, i.e., memristance jumps in M-C filter, a transfer function cannot be used to describe the filter output. As shown in Figure 2 if memristance jumps from \(M_k\) to \(M_{k+1}\) or \(M_{k+1}\) to \(M_{k}\), the circuit behaves nonlinearly even in steady state. Since transfer function in frequency domain cannot be used to analyze the nonlinear operation, simulations are used to show the performances of M-C filters in time domain. State-space model of memristor based LP and HP filters can be given as

\[
i_{\text{Mem}} = \frac{dq_{\text{Mem}}}{dt} = \frac{v_i - v_C}{M(q)} \quad (14)
\]

\[
\frac{dv_C}{dt} = \frac{1}{C} i_{\text{Mem}} = \frac{i_{\text{Mem}}}{C} = \frac{v_i - v_C}{M(q)C} \quad (15)
\]

Since memristance is a nonlinear function, the state space system is simulated using Simulink™ toolbox of MATLAB™. Its block diagram is shown in Figure 5. For M-C LP filter, the output voltage is the capacitor voltage \(v_C\) and for M-C HP filter, the output voltage is the memristor voltage \(v_{\text{Mem}}\). Simulations are done by varying initial memristor charge, input voltage amplitude, and input voltage frequency.

5. Simulation Results

In this section, memristor based LP and HP filters are simulated for different frequencies, amplitude and initial memristor charge values. Current-voltage curves memristance, memristor charge, gain and THD characteristics of the simulated filters are observed.

![Simulink block diagram of M-C filter](Figure 5)
5.1.1. Memristor Based Low-Pass Filter

A sinusoidal input voltage of $v_i(t) = V_m \times \sin(\omega t)$ is applied to the filter as shown in Figure 4 (b). If $V_m$ is high enough and initial charge is near the break points in the $\phi-q$ curve, memristive switching in the M-C filter occurs. By choosing $V_m = 0.5 \text{ V}$ and $q_0 = 0.9\times10^{-6} \text{ C}$, memristive switching is observed as shown in Figure 6. Input and output voltages of the filter are shown in Figure 6 (a). Memristance takes two different values either as 1000 or 1500 ohms as shown in Figure 6 (b). Variation of memristance causes a distortion on memristor current and voltage as shown in Figure 6 (c). The zero-crossing i-v hysteresis loop clearly indicates that the memristive switching occurs as shown in Figure 6 (d).

If the amplitude of the input voltage decreases and the initial charge gets away from the break-points, memristor behaves as if a resistor. Such a behavior is obtained by taking $V_m = 0.1 \text{ V}$ and $q_0 = 0.7\times10^{-6} \text{ C}$. Memristive switching does not occur for this parameter set and memristance remains constant. In this situation, characteristic of the M-C filter is similar to the traditional R-C filter as shown in Figure 7.

![Figure 6. LP filter in MS case for $V_m = 0.5 \text{ V}$, $C = 1 \mu\text{F}$, $f = 10 \text{ Hz}$, $q_0 = 0.9\times10^{-6} \text{ C}$. (a) Input and output voltages, (b) memristor charge and memristance, (c) memristor voltage and memristor current (d) I-V hysteresis curve of memristor.](image-url)
Figure 7. LP filter in no MS case for $V_m = 0.1 \text{ V}$, $C = 1 \mu\text{F}$, $f = 10 \text{ Hz}$, $q_0 = 0.7 \times 10^{-6} \text{ C}$. (a) Input and output voltages of the filter, (b) memristor charge and memristance, (c) memristor voltage and memristor current (d) I-V hysteresis curve of memristor.

5.1.2. Memristor Based High-Pass Filter

M-C HP filter which has formed as shown in Figure 4 (d) is considered. If $V_m$ is high enough and initial charge is near the break points in the $\phi$-$q$ curve, memristive switching in the M-C filter occurs. By choosing $V_m = 0.5 \text{ V}$ and $q_0 = 0.95 \times 10^{-6} \text{ C}$, memristive switching is observed as shown in Figure 8. Input and output voltages are shown in Figure 8 (a). Memristance takes two different values either as 1000 or 1500 ohms as shown in Figure 8 (b). Variation of memristance causes a distortion on memristor current and voltage as shown in Figure 8 (c). The i-v hysteresis loop clearly indicates that the memristive switching occurs as shown in Figure 8 (d).

Figure 8. HP filter, in MS case for $V_m = 0.5 \text{ V}$, $C = 1 \mu\text{F}$, $f = 1 \text{ kHz}$, $q_0 = 0.95 \times 10^{-6} \text{ C}$. (a) Input and output voltages, (b) memristor charge and memristance, (c) memristor voltage and memristor current (d) I-V hysteresis curve of memristor.
Similar to LP filter, if the amplitude of the input voltage and/or the initial charge decrease, memristor behaves as if a resistor. Such an operation is obtained defining as $V_m = 0.1 \text{ V}$ and $q_0 = 0.7 \times 10^{-6} \text{ C}$. Memristive switching does not occur for this parameter set and memristance remains constant. In this situation, characteristic of the M-C filter is similar to the traditional R-C one as shown in Figure 9.

**Figure 9.** HP filter, no MS case $V_m = 0.1 \text{ V}$, $C = 1 \mu\text{F}$, $f = 1\text{kHz}$, $q_0 = 0.7 \times 10^{-6} \text{ C}$ (a) Input and output voltages, (b) memristor charge and memristance, (c) memristor voltage and memristor current (d) I-V hysteresis curve of memristor.

5.2. Analysis of Gain Characteristics of Filters

Gain characteristics of M-C LP and HP filters are obtained as a function of frequency and memristor charge in this section using simulations.

**Figure 10.** LP filter gain response for $V_m = 0.5 \text{ V}$, $C = 1 \mu\text{F}$. (a) Three different initial charge ($q_0$) values and varying frequency from 1 Hz to 100 kHz (b) for $f = 1\text{Hz}$ and varying initial charge from $-2.5 \times 10^{-6} \text{ C}$ to $+2.5 \times 10^{-6} \text{ C}$.

5.2.1. Memristor Based Low-Pass Filter

It is observed that, the filter gains for $q_0 = -q$ and $q_0 = +q$ values are the same and a little bit lower than the value at $q_0 = 0$ at same frequency as shown in Figure 10 (a). This situation is seen in Figure 10 (b) more clearly by varying initial charge $q_0$ from $-2.5 \times 10^{-6} \text{ C}$ to $+2.5 \times 10^{-6} \text{ C}$.
5.2.2. Memristor Based High-Pass Filter

It is observed that, the filter gain at \( q_0 = -q \) and \( q_0 = +q \) value is the same since the memristance is an even function and a little bit lower than the value of \( q_0 = 0 \) at same frequency as shown in Figure 11 (a). This situation can be seen from the Figure 11 (b) more clearly.

As shown in Figure 11 (b), the filter gain is not a steep five-step PWL function as defined in (13). The filter gain as shown in Figure 11 (b) is smoothed somewhat. The reason is memristive switching occurring. In other words, the input magnitude and/or input frequency and/or the memristor initial charge result in varying gain. Combination of them defines at which segments the memristor operates for how long and thus the filter gain.

5.3. Analysis of Total Harmonic Distortion of the Filters

5.3.1. Memristor Based Low-Pass Filter

THD of the filter analyzed versus frequency at \( q_0 = 0 \) Figure 12 (a). THD is low throughout the operating frequency range and starts increasing at high frequencies. The effect of memristive switching on THD can be clearly seen from the Figure 12 (b). THD increases with the initial memristor charge values near to the break points. However it is still low at low operating frequencies.

Figure 11. HP filter gain response for \( V_m = 0.5 \) V, \( C = 1 \) µF. (a) Three different initial charge \( (q_0) \) values and varying frequency from 1 Hz to 100 kHz (b) for \( f = 1 \) kHz and varying initial memristor charge from \(-2.5 \times 10^{-6} \) C to \(+2.5 \times 10^{-6} \).

Figure 12. Total Harmonic Distortion of the memristor based LP filter versus (a) frequency (at \( q_0 = 0 \)) and (b) memristor initial charge (at \( f = 1 \) Hz) for \( V_m = 0.5 \) V, \( C = 1 \) µF.
5.3.2. Memristor Based High-Pass Filter

THD of the filter analyzed versus frequency at the memristor initial charge, \( q_0 = 0 \) Figure 13 (a). THD is low throughout the operating frequency range. However as the specific characteristics for memristor, the effect of memristive switching on THD at 1 KHz can be clearly seen from the Figure 13 (b). THD increases at the charge values near the break points. However it is still low throughout operating frequency levels.

THD varies with respect to initial memristor charge. Distortion becomes maximum around breaking charge points. It is not monotonous. Also when the initial memristor charge is kept constant as show in Figure 13 (a), THD first decreases, reaches to a minimum point starts increasing again, reaches a maximum point and start decreasing again. That means for different operating frequencies, the distortion may increase or decrease due to memristive switching. This property of M-C HP filter has not been reported in literature before.

![Graph](image)

Figure 13. Total Harmonic Distortion of the memristor based HP filter versus (a) frequency (at \( q_0 = 0 \)) and (b) memristor initial charge (at \( f = 200 \text{ Hz} \)) for \( V_m = 0.5 \text{ V}, C = 1 \mu\text{F} \).

6. Conclusions

In this paper, by using generalized n-segment PWL modeling of memristor, M-C LP and HP memristor-based analog filters are investigated. The characteristics of filters are observed by using simulations. The effect of memristive switching on LP and HP memristive filters are analyzed. Total harmonic distortion is also obtained as a function of frequency and memristor initial charge. It has been found out that the memristive switching in M-C filters results in distortion at output voltage waveforms. Thus, memristive filters produce harmonics. These issues are for the first time reported in memristor based filters.

It is expected that, memristors can be applied and provide new additional features to analog circuit applications. With the results of this study we observed some important concerns designing memristor based analog circuits. To design filters with memristors, THD of filter in operation range or distortion of the filter must be minimized to prevent reduction of the performance. We have also found that, the filter distortion may not be monotonous and that’s why simulation methods can be employed to design memristive filters due to their nonlinearity.

7. References


Şuayb Ç. YENER was born in Sakarya, Turkey in 1982. He received B.Sc. degree from Sakarya University in Electrical and Electronics Engineering in 2004 and M.Sc. degree from Istanbul University in Electronics and Communication Engineering in 2007, respectively. He is currently a Ph.D. student in Istanbul Technical University, and also a research assistant in Sakarya University. His main research interests are CMOS analog circuits, low-power analog circuit design, circuit and electronic device modeling, modeling of the memristor and memristor based circuits.
Reşat Mutlu was born in Tekirdağ, Turkey in 1973. He received B.Sc. degree from YILDIZ Teknik University in Electrical Engineering in 1995 and M.Sc. and Ph.D. degree from Rensselaer Polytechnic Institute in Electric Power Engineering in 1998 and 2004, respectively. He is currently an assistant professor at Electronics and Telecommunication Engineering Department, Namik Kemal University, Tekirdağ, Turkey. His main research interests are modeling of memristor and memristive systems, analog and computer memory applications of memristive systems, analysis and modeling of memcapacitors, teaching of the memristive systems.

Hakan KUNTMAN received his B.Sc., M.Sc. and Ph.D. degrees from Istanbul Technical University in 1974, 1977 and 1982, respectively. In 1974 he joined the Electronics and Communication Engineering Department of Istanbul Technical University. Since 1993 he is a professor of electronics in the same department. His research interest includes design of electronic circuits, modeling of electron devices and electronic systems, active filters, design of analog IC topologies. Dr. Kuntman has authored many publications on modeling and simulation of electron devices and electronic circuits for computer-aided design, analog VLSI design and active circuit design. He is the author or the co-author of 106 journal papers published or accepted for publishing in international journals, 163 conference papers presented or accepted for presentation in international conferences, 156 Turkish conference papers presented in national conferences and 10 books related to the above mentioned areas. Furthermore he advised and completed the work of 9 Ph.D. students and 39 M.Sc students. Currently, he acts as the advisor of six Ph.D. students. Dr. Kuntman is a member of the Chamber of Turkish Electrical Engineers (EMO).